Differential effect of the pension system on education and income by life expectancy

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Abstract

This paper investigates the differential impact that alternative pension systems have on the labor supply and the accumulation of physical and human capital for individuals that differ by their learning ability and levels of life expectancy. Our analysis is calibrated to the US economy using a general equilibrium model populated by overlapping generations, in which all population groups interact through the pension system, the labor market, and the capital market.

1 Motivation

In many countries, economic development has been accompanied with significant increases in life expectancy. Since the 19th century life expectancy has increased by forty years at a rate of three months per year (Oeppen and Vaupel (2002) and Lee (2003)). As noted in Pestieau and Ponthire (2012) the increase in longevity has not been accompanied by a decrease in the variability of longevity across individuals. Indeed, Pestieau and Ponthire (2012) argue that "Longevity inequalities across groups within nations may be as large - if not larger - than longevity inequalities between nations."

How far may such inequalities in life span translate into differences of life cycle decisions? Moreover, may social institutions intervene with the heterogenous life spans in a society and thereby affect the inequality within the economy? As demonstrated in Pestieau and Ponthire (2012) with a benchmark OLG model with two periods and two types of agents (long and short lived), a utilitarian social planner who maximizes the sum of individual utilities may thereby induce a redistribution from the short to the long lived individuals.

Existing theoretical models that analyze the effect of mortality on education and labor supply are numerous but partly contradict the historical empirical evidence such as the decline in labor supply with increasing longevity. The positive link between human capital investment and life expectancy as observed in historical time series is also theoretically replicated through the well-known Ben-Porath (1967) mechanism and overall undisputed (de la Croix and Licandro (1999), Kalemli-Ozcan et al. (2000), Zhang et. al. (2001), Zhang et. al. (2003), Cervellati and Sunde (2005), Soares (2005), Zhang and Zhang (2005), Jayachandran and Lleras-Muney (2009), and Oster et al. (2013)), except by Hazan and Zoabi (2006). However, studies about the effect of mortality improvements on the decline in labor supply are scarce and offer several complementary explanations. For instance, Kalemli-Ozcan and Weil (2010) suggest that the decline in labor supply might be explained by reductions in the risk of dying before retirement, named "uncertainty effect". More recently, Bloom et al. (2014) point out that positive income effects along the 20th century might have offset the gains in healthy life after retirement, or "compression of morbidity" effect (see Bloom et al. (2007)). Several authors have recently shown that the link between life expectancy and labor supply depends on the age pattern of mortality improvements. In particular, mortality declines during adulthood may cause early retirement, while reductions in mortality at older ages delay retirement (d'Albis et al. (2012) and Strulik and Werner (2012)).

However the role of heterogeneous life spans on the education and labor supply decisions in a society has not yet been investigated. In our paper we study how the composition of the population in low and high life span people may result in different aggregate levels of education and labor supply as well impact on the inequality in a society. To do so, we implement two alternative pension benefit formulae and study how far such a welfare institutional arrangement may reduce or enhance the resulting inequality in a society.

Our results indicate that the pension system may enhance income inequality in the long run. In particular, we show that larger inequality can be caused by a non progressive pension system, such as those in many European countries, because of the positive effect that the pension system has on the marginal benefit of education.

The paper is organized as follows: Section 2 introduces the model setup. In Section 4, we explain the thought experiments. In Section 5 we solve the model numerically and consider the role of different pension systems for behavioral effects, redistributive effects and macroeconomic effects in detail. Section 6 concludes the paper.

2 Model

This paper builds a computable overlapping generation model of labor supply, human capital formation, and physical capital accumulation with heterogeneity in human capital and life expectancy. The model analyzes the economic consequences of running a pension system with a progressive replacement rate versus a flat replacement rate in the context of within cohort heterogeneity in life expectancy and human capital. The model is close in spirit to Heckman et al. (1998), Ludwig et al. (2012) and more recently to Geppert (2015). Nevertheless, our focus is on the analysis of different pension systems, rather than about rising wage inequality (Heckman et al., 1998) or the wealth inequality driven by the demographic change (Geppert, 2015). Moreover, the previous models are extended by introducing longevity differentials within the same cohort.

2.1 Demographics

Time is discrete. Individuals face mortality risk and may live up to a maximum of 120 years. We assume agents born are heterogenous by their frailty level and their learning ability level, denoted by the letter θ. Individuals belong to any of three different frailty groups, which are distinguished by the variable $\mu \in \{1, 2, 3\}$. The first group ($\mu = 1$) is assumed to be the most frail and has the shortest longevity, individuals that belong to the second group ($\mu = 2$) have an average frailty and hence their life expectancy is close to the average of the whole population, individuals belonging to the third group ($\mu = 3$) are the less frail and have the longest life expectancy. Let the probability of surviving to age j in year t of an individual of type μ be

$$s_{t,j}(\mu) = \prod_{u=0}^{j-1} \pi_{t-j+u,u}(\mu) \text{ with } s_{\cdot,0}(\mu) = 1 \text{ and } s_{\cdot,\Omega}(\mu) = 0. \tag{1}$$

where $\pi_{t,u}(\mu)$ is the conditional probability of surviving to age u for an individual in year t that belongs to the frailty group μ.

Let $N_{t,j}(\theta, \mu)$ be the number of people of type (θ, μ) who are j years old at time t. Let $G(\theta, \mu)$ be the joint distribution of individuals of type (θ, μ) at birth. We assume for simplicity that our population is closed to migration and that individuals belonging to each demographic group (θ, μ) stay in that group until death. The dynamics of the population group (θ, μ) is described by the following set of equations:

$$N_{t+1,0}(\theta, \mu) = s_{t+1,0}(\mu) \sum_{j=0}^{\Omega-1} F_{t,j} \frac{N_{t,j}(\theta, \mu) + N_{t,j+1}(\theta, \mu)\pi_{t,j+1}(\mu)}{2} f_{fab}, \quad (2)$$

$$N_{t+1,j+1}(\theta, \mu) = N_{t,j}(\theta, \mu)\pi_{t,j}(\mu) \quad (3)$$

where $F_{t,j}$ is the fertility rate at time t for an individual of age j and f_{fab} is the standard frequency of female at birth (i.e. $f_{fab} = 0.4886$). Eq. (2) is the total number of surviving births in year $t+1$, whereas Eq. (3) accounts for the total number of survivors of type (θ, μ) to time $t+1$ for the cohort born in year s. The total population size in year t equals $N_t = \sum_{j=0}^{\Omega} N_{t-j,0} \int s_{t,j}(\mu) dG(\theta, \mu)$.

2.2 Firms

Firms operate in a perfectly competitive environment and produce one homogeneous good, which can be consumed or stored by individuals, according to a Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \left(A_t H_t\right)^{1-\alpha}, \quad (4)$$

where α is the capital share, Y_t is the output in period t, K_t is the stock of physical capital in period t, A_t is the labor-augmenting technological progress, and H_t is the aggregate stock of employed human capital in period t. For simplicity, we assume A_t increases annually at a constant rate g_A. It is important to realize that a Cobb-Douglas production function rather than a CES production function is justified in this framework to avoid endogenous economic growth when the elasticity of substitution between factors is high (Barro and Sala-i-Martin, 2003).

Capital stock evolves according to the law of motion $K_{t+1} = K_t(1-\delta) + I_t$, where δ is the depreciation rate of capital and I_t is aggregate gross investment. Production factors are paid at their marginal products:

$$R_t^H = (1-\alpha)\left(Y_t/H_t\right), \quad (5)$$

$$r_t = \alpha \left(Y_t/K_t\right) - \delta. \quad (6)$$

where R_t^H is the rental rate on human capital at time t and r_t is the net return on physical capital at time t.

2.3 Household's problem

Households at time t are heterogeneous in seven dimensions: age (j), asset holdings (a), the stock of human capital (h), the average past pension earnings (p), the household size (η) –measured in units of equivalent adult consumption, the learning ability (θ), and the frailty group (μ). Let denote the set of the state variables at age j at time t for an agent of type (θ, μ) by $X_{t,j}(\theta, \mu) = \{a_{t,j}, h_{t,j}(\theta), p_{t,j}, \eta_{t,j}(\mu), \theta, \mu\}$. The expected utility ($V$) of a household head of type (θ, μ) and age j at time t takes the following functional form:

$$V_{t,j}\left(X_{t,j}(\theta, \mu)\right) = U\left(c_{t,j}, z_{t,j}\right) + \beta\pi_{t+1,j+1}(\mu)V_{t+1,j+1}\left(X_{t+1,j+1}(\theta, \mu)\right) \tag{7}$$

where β is the subjective discount factor, U is the period utility function (with $U_c \geq 0, U_z \geq 0, U_{cz} \leq 0, U_{cc} \leq 0,$ and $U_{zz} \leq 0$), $c_{s,t}$ is the consumption of the household head, and $z_{s,t}$ is the leisure time of the household head at age j in time t.

We assume agents start making decisions at the age of 15, which corresponds to the age after nine years of compulsory education. Each period, agents are endowed with one unit of time. They optimally choose their consumption path, leisure time, hours of work, and the fraction of time invested in human capital formation. In this model we can distinguish up to three periods of human capital investment: i) a period of specialization in schooling, ii) a period of on-the-job training in which the time devoted to human capital investment monotonically decreases until retirement, and iii) a retirement period in which individuals do not invest in human capital (Blinder and Weiss, 1976). Agents start with zero assets, zero pension earnings, and an initial human capital h_0 that is similar for all individuals regardless their life expectancy and ability due to the compulsory education (i.e. $a_{t,15} = p_{t,15} = 0$ and $h_{t,15}(\theta) = h_0$ for all t). Similar to Ríos-Rull (2001), Braun et al. (2009), and Ludwig et al. (2012) agents may borrow against future labor income in order to finance their consumption and their educational investment. Thus, assets held (a) evolves over the life cycle according to

$$a_{t+1,j+1} = \begin{cases} R_t(a_{t,j} + \mathrm{tr}_{t,j}(\theta, \mu)) + (1-\tau_t)y_{t,j} - c_{t,j}\eta_{t,j}(\mu) & \text{for } j \leq J_R, \\ R_t(a_{t,j} + \mathrm{tr}_{t,j}(\theta, \mu)) + b_{t,j}(J_R) - c_{t,j}\eta_{t,j}(\mu) & \text{for } j > J_R, \end{cases} \tag{8}$$

where $R_t = 1 + r_t$ is the capitalized rate of return of capital at time t, $\mathrm{tr}_{t,j}(\theta, \mu)$ is the average bequest received at age j at time t by an agent of type (θ, μ), τ_t is the social security contribution rate in year t, $y_{t,j} = R_t^H h_{t,j}\ell_{t,j}$ is the (gross) labor income at age j in year t, which is a function of the rental rate of human capital in year t (R_t^H), the stock of human capital at age j in year t

($h_{t,j}$), and the fraction of time devoted to work ($\ell_{t,j}$); $\eta_{t,j}(\mu)$ is the household size measured in terms of equivalent adult consumption. This assumption has been shown to be important for explaining household saving (Curtis et al., 2015; Attanasio and Weber, 2010; Browning and Ejrnaes, 2009; Browning and Lusardi, 1996). J_R is the retirement age, and $b_{t,j}(J_R)$ is the pension benefits received at age j in year t by an individual retired at age J_R, which can be decomposed in the following three components:

$$b_{t,j}(J_R) = \lambda(J_R)\psi(p_{t,j})p_{t,j}, \tag{9}$$

where $\psi(p_{t,j})$ is the replacement rate associated to $p_{t,j}$ pension earnings and $\lambda(J_R)$ is the pension penalties or rewards from retirement at age J_R.

The introduction of pension earnings as a state variable in Eqs. (7)-(8) implies that agents understand the rules on how pension benefits are calculated (Ludwig and Reiter, 2010; Sanchez-Romero et. al., 2013). In other words, agents internalize that higher labor earnings affects positively on their future pension benefits. In particular, pension earnings evolve over time according to

$$p_{t+1,j+1} = \begin{cases} I_t p_{t,j} + \varrho_j y_{t,j}(\theta) & \text{for } j \leq J_R, \\ p_{t,j} & \text{for } j > J_R, \end{cases} \tag{10}$$

where I_t is a weight factor on past pension earnings and ϱ_j is the weight of labor income at age j on pension earnings.

Agents may devote time to get education, denoted by e, to increase their future human capital and hence labor income. We assume human capital accumulates according to a standard Ben-Porath (1967) technology

$$h_{t+1,j+1}|\theta = \begin{cases} h_{t,j}(1-\delta_h) + q\left(h_{t,j}, e_{t,j}; \theta\right) & \text{for } 15 \leq j \leq J_R, \\ h_{t,j}(1-\delta_h) & \text{for } j > J_R. \end{cases} \tag{11}$$

That is, human capital increases due to investments in human capital $q(h,e)$ (with $q_h \geq 0, q_e \geq 0, q_{he} \leq 0, q_{hh} \leq 0$, and $q_{ee} \leq 0$), and decreases due to the depreciation of human capital at an annual rate δ_h. We use a standard Ben-Porath human capital production function

$$q\left(h_{t,j}, e_{t,j}; \theta\right) = \varphi(\theta)\left(h_{t,j}e_{t,j}\right)^{\gamma} \text{ with } \varphi > 0, \gamma \in (0,1), \tag{12}$$

where $\varphi(\theta)$ is the learning ability for the individual of type θ and γ is the returns to scale in human capital investment.

2.3.1 Household's decision problem

Households optimally allocate the resources by maximizing (7) with respect to consumption, leisure, and human capital investment subject to (8)-(11) and the time constraints $z_{t,j} \geq 0, \ell_{t,j} \geq 0, e_{t,j} \geq 0$, and $z_{t,j} + \ell_{t,j} + e_{t,j} = 1$. All feasible solutions are derived in a technical appendix. In this section, we focus on the pre-retirement period.[1]

The following first-order conditions govern the model:

$$U_{c_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} \eta_{t,j}(\mu), \tag{13}$$

$$U_{z_{t,j}} \geq \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H h_{t,j}, \tag{14}$$

and

$$\frac{\partial V_{t+1,j+1}}{\partial h_{t+1,j+1}} \gamma \varphi(\theta) h_{t,j} (h_{t,j} e_{t,j})^{\gamma-1} \geq \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H h_{t,j}, \tag{15}$$

(the marginal benefit of education is equal, or greater, than the marginal cost)

A strict inequality implies that agents specialize in schooling and devote all their time between schooling and leisure.

$$\frac{\partial V_{t,j}}{\partial a_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} R_t, \tag{16}$$

(intertemporal arbitrage in returns on physical capital)

$$\frac{\partial V_{t,j}}{\partial h_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \left(\frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} (1 - \tilde{\tau}_{t,j}) R_t^H \ell_{t,j} + \frac{\partial V_{t+1,j+1}}{\partial h_{t+1,j+1}} r_{t,j}^h(\theta) \right), \tag{17}$$

(the marginal value of human capital is the return to current and future earnings)

$$\frac{\partial V_{t,j}}{\partial p_{t,j}} = \beta \pi_{t+1,j+1}(\mu) \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} I_t, \tag{18}$$

(the marginal value of pension earnings is the return to future pension earnings)

where $r_{t,j}^h(\theta) \equiv \frac{\partial h_{t+1,j+1}}{\partial h_{t,j}} = \gamma \varphi(\theta) e_{t,j} (h_{t,j} e_{t,j})^{\gamma-1} + 1 - \delta_h$ is the rate of return to human capital at age j in year t for an agent of type θ, and $\tilde{\tau}_{t,j}$ is the effective social security tax rate, which is given by

$$\tilde{\tau}_{t,j} = \tau_t - \rho_j \left. \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} \middle/ \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}} \right. .$$

[1] During retirement our agents only decides about their consumption path. Leisure is equal to one, and therefore both the hours worked and the human capital investment are equal to zero.

Hence, agents perceive only part of the social security contribution rate as a tax and it can even be seen as a subsidy when $\tau_t \leq \rho_j \, \frac{\partial V_{t+1,j+1}}{\partial p_{t+1,j+1}} \Big/ \frac{\partial V_{t+1,j+1}}{\partial a_{t+1,j+1}}$.

From (13) and (16) the standard optimal consumption path for an individual of type μ is given by

$$\frac{U_{c_{t,j}}}{U_{c_{t+1,j+1}}} = \left(\frac{\eta_{t,j}(\mu)}{\eta_{t+1,j+1}(\mu)} \right) \beta \pi_{t+1,j+1}(\mu) R_{t+1}, \tag{19}$$

where Eq. (19) is the well-known Euler condition. The left-hand side of Eq. (19) is the marginal rate of substitution between present and future consumption. The term in parenthesis on the right-hand side of (19) accounts for the evolution of the household size. Thus, when the household size increases the consumption of the household head decreases, and vice versa. While the second term on the right-hand side of (19) implies that, in the absence of an annuity market, consumption increases when agents discount future consumption less than the market, i.e. $\beta \pi_{t+1,j+1}(\mu) R_{t+1} > 1$, and decreases when $\beta \pi_{t+1,j+1}(\mu) R_{t+1} < 1$. Thus, for the same ability level, the existence of three frailty groups in the population implies three different consumption/saving trajectories.

The optimal labor supply decision is characterized by the marginal rate of substitution between consumption and leisure:

$$\text{MRS}_{c,z} \equiv \frac{U_{z_{t,j}}}{U_{c_{t,j}}} \geq \frac{(1-\tilde{\tau}_{t,j}) R_t^H h_{t,j}}{\eta_{t,j}(\mu)} \qquad \text{for } j \in \{15, \ldots, J_R\}. \tag{20}$$

Eq. (20) implies that leisure is increasing the larger is the net (of implicit labor tax) wage rate per hour worked relative to the household consumption.

From (12) and (15)-(17), the optimal on-the-job training investment satisfies

$$e_{t,j} = \frac{1}{h_{t,j}} \left(\gamma \varphi(\theta) \sum_{x=j+1}^{J_R} \left[\prod_{z=j+1}^{x} \frac{r^h_{t-j+z,z}(\theta)}{R_{t-j+z}} \right] \frac{1-\tilde{\tau}_{t-j+x,x}}{1-\tilde{\tau}_{t,j}} \frac{R^H_{t-j+x}}{R^H_t} \frac{\ell_{t-j+x,x}}{r^h_{t-j+x,x}(\theta)} \right)^{\frac{1}{1-\gamma}}, \tag{21}$$

According to (21) the optimal fraction of time devoted to on-the-job training is increasing the higher is the learning ability of the individual, the larger the future hours worked, the higher the retirement age (J_R), and the lower the future returns on physical capital. Moreover, it is worth stressing that the pension system may also influence the optimal investment in education through the effective social security tax rate. Thus, in this framework, there exists a positive relationship between on-the-job training and the pension system when future effective social contribution rates are lower than present effective social contributions rates, i.e. $\tilde{\tau}_{t-j+x,x} < \tilde{\tau}_{t,j}$ for $j \leq x \leq J_R$. Assuming that neither the social contribution rate (τ) nor the weight of labor income on pension earnings (ρ) change over time, the pension system has an unambiguous positive effect on human capital investment if $\prod_{v=j+1}^{x} \frac{I_{t-j+v}}{R_{t-j+v}} < 1$, and it is negative otherwise. Moreover, the incentive to invest in human capital increases, the greater is the gap between the capitalized returns to physical capital and the weighting factor on past pension earnings, all other variables constant.

2.4 Government

We assume the government has two main activities: i) running a balanced pay-as-you-go (PAYG) pension system and ii) levying a 100% tax on assets held by agents dead each year and distribute it among the surviving population.

To guarantee a zero deficit in the PAYG pension system, the government is assumed to modify the social security contribution rate τ_t each year in order to finance all the pension benefits claimed by retirees. The social contribution rate is the same for all type of agents regardless their age, learning ability, or life expectancy. Moreover, the government sets a mandatory retirement age J_R, which it is assumed to be the same in all periods.

To be as realistic as possible, we assume the government distributes the bequest among individuals belonging to the same population type (θ, μ). Moreover, as a general rule, the bequest is given to the surviving members of the generation prior to the individual dying, which closely coincides with the mean-age of their children.[2] However, when deaths occur below age 42, the bequest is assumed to be distributed within the same age group (i.e. cohort) since by assumption individuals under age 15 cannot hold assets. Thus, the amount of bequest received at age j at time t by an individual of type (θ, μ) is

$$\mathrm{tr}_{t,j}(\theta,\mu) = \begin{cases} 0 & \text{If } j < 15, \\ a_{t,j+28}(\theta,\mu)\frac{N_{t,j+28}(\theta,\mu)(1-\pi_{t,j+28}(\mu))}{N_{t,j}(\theta,\mu)\pi_{t,j}(\mu)} + a_{t,j}(\theta,\mu)\frac{1-\pi_{t,j}(\mu)}{\pi_{t,j}(\mu)} & \text{If } 15 \leq j \leq 42, \\ a_{t,j+28}(\theta,\mu)\frac{N_{t,j+28}(\theta,\mu)(1-\pi_{t,j+28}(\mu))}{N_{t,j}(\theta,\mu)\pi_{t,j}(\mu)} & \text{If } j > 42. \end{cases} \tag{22}$$

Realize that Eq. (22) is a generalization of the simple model proposed by

[2]The gap between each generation is assumed to be 28 years, which coincides with the mean age of childbearing.

Piketty (2011) to assess the evolution inheritances received by each age group over time.

3 Equilibrium conditions under perfect foresight

By assuming perfect foresight, agents will differ because of their learning ability (θ), frailty (μ), and the different prices over their life course. Let P_s be the vector of rental prices of physical capital and human capital, and social security contribution rates faced by an individual born in year s over the lifecycle.

Given initial values $\{\alpha, g_A, \delta, \varphi(\theta), \gamma, \mu\}$, we can define the recursive competitive equilibrium as the sequence of a set of households policy functions $\{X_{t,j}(\theta,\mu), c_{t,j}, z_{t,j}, e_{t,j}\}$, government policy functions $\{B_{t,j}, \tau_t, \lambda(J_R), \psi(p)\}$, and factor prices $\{R_t^H, r_t\}$, for $j \in \{15, \ldots, \Omega\}$ and $t > 0$, such that

1. Given the factor prices and government policy functions, household policy functions satisfy Eqs. (13)-(18)
2. Factor prices equal their marginal productivities so that Eqs. (5) and (6) hold.
3. The government's budget constraints

$$\tau_t R_t^H H_t = \sum_{j=J_R+1}^{\Omega} N_{t-j,0} \int b_{t,j}(J_R; \theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta,\mu), \tag{23}$$

(Total contributions paid equal total pension claimed)

and

$$\sum_{j=15}^{\Omega} N_{t-j,0} \int tr_{t,j}(\theta,\mu) s_{t,j}(\mu) \pi_{t,j}(\mu) dG(\theta,\mu) = \sum_{j=15}^{\Omega} N_{t-j,0} \int a_{t,j}(\theta,\mu,P_{t-j}) s_{t,j}(\mu)(1-\pi_{t,j}(\mu)) dG(\theta,\mu)$$

(Total wealth transfers received equal total wealth transfers given) (24)

are satisfied.

4. The aggregate stock of physical capital and the aggregate stock of employed human capital are given by:

$$K_t = \sum_{j=15}^{\Omega} N_{t-j,0} \int a_{t,j}(\theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta, \mu), \tag{25}$$

$$H_t = \sum_{j=15}^{\Omega} N_{t-j,0} \int h_{t,j}(\theta, \mu, P_{t-j}) \ell_{t,j}(\theta, \mu, P_{t-j}) s_{t,j}(\mu) dG(\theta, \mu). \tag{26}$$

5. The commodity market clears:

$$Y_t = C_t + S_t, \tag{27}$$

where $C_t = \sum_{j=15}^{\Omega} N_{t-j,0} \int c_{t,j}(\theta, \mu, P_{t-j}) \eta_{t,j}(\mu) s_{t,j}(\mu) dG(\theta, \mu)$ is the total consumption in year t and S_t is the gross saving in year t.

4 Parametrization

In this paper we aim at studying the economic consequences of running a PAYG pension system with a progressive replacement rate versus a flat replacement rate.

We focus our analysis to the US economy, since the Old-Age, Survivors, and Disability Insurance (OASDI), which is the largest component of the US Social Security, runs a pension system with a progressive replacement rate (Golosov et al., 2013). A recent study applied to the US by Ludwig et al. (2012) has shown that adding endogenous human capital accumulation in OLG models significantly reduces the adverse welfare consequences of population aging. Thus, in order to introduce endogenous human capital and have agents with different labor income histories –and hence different replacement rates at retirement, we assume individuals differ by their learning ability. As a consequence, individuals with higher abilities will accumulate more human capital and have higher incomes. In addition, since many studies have found a positive correlation between schooling and health, we introduce additional heterogeneity by life expectancy. This is done by assuming that individuals may belong to different mortality frailty groups (Vaupel et. al., 1979). Moreover, following Lleras-Muney (2005) we set the population structure such that there exists a negative correlation between ability and frailty. As a result, individuals with longer schooling also have on average longer life expectancy.

To keep the model as tractable as possible, we assume individuals are assigned at birth to any combination of three possible learning abilities and three frailty groups. This gives a total of nine different population groups within each cohort. The reason for having three cases in each characteristic rather than two is to be able to analyze the average individual at the same time as having greater heterogeneity within a computationally tractable model.

In this paper we run two alternative scenarios. A baseline that replicates the current parametric components of the US pension system and second, a counterfactual experiment with a pension system that applies a flat replacement rate to all retirees, whose value correspond to the average replacement rate of the total retired population in the baseline scenario. Finally, since we are also interested in understanding the effect of the pension system on the economic decision making of our nine different population groups, we have intentionally abstracted from the introduction of public goods and services, which would be financed by general taxes that may have an influence on our endogenous variables.

Next, we explain the main assumptions introduced in the demographic set up and in the economic model to disentangle the different effects.

4.1 Demographics

We replicate the overall demographic features of the US population from year 1850 to 2010 – that is, single age-specific fertility rates and single age-specific mortality rates – using inverse population projection methods (Lee, 1985; Oeppen, 1993). The demographic information for the period 1850-2010 is taken from the US Census Bureau, the Human Mortality Database (HMD), and the Human Fertility Database (HFD). Future fertility rates are based on UN assumptions for the US population, whereas the future average mortality is calculated based on the Lee-Carter model (Lee and Carter, 1992).[3] Before year 1800 we assume a stable population (i.e. constant population growth). At the individual level, we assume recently born individuals differ by their learning

[3] According to the Lee-Carter model, the temporal component of mortality gains at time t can be described, on average, according to the following time series

$$k_{t+1} = \hat{\mu} + \phi k_t. \tag{28}$$

In order to introduce the heterogeneity in our model, we assume individuals are subject to different drifts as follows

$$\hat{\mu} = \bar{\mu} + \mu \text{ with } \mu \sim \mathcal{U}(-\bar{\mu}, \bar{\mu}) \text{ and } 0 < \mu < \bar{\mu}. \tag{29}$$

Thus, $\bar{\mu}$ matches the observed drift in the population, whereas each population group would have a different μ value, and hence a different life expectancy. We apply this model from year 1900 onwards to the US population.

ability and life expectancy. We consider three learning abilities θ ={*low=1, average=2, high=3*} and three alternative life expectancies μ ={*high=1, average=2, low=3*}, with equal marginal probabilities. Nevertheless, the population weights, shown in Table 1, are set to capture the positive correlation between life expectancy and learning ability.

[Table 1 about here.]

Figure 1 shows the evolution of the main aggregate demographic variables: life expectancy, total fertility rate, and population distribution for several years. Panel 1(a) shows the evolution of the life expectancies by frailty group. Our simulated frail populations, which do not pretend to replicate actual data from the US, present an increasing gap between the life expectancies of the different groups. In particular, we assume, in our hypothetical populations, that the life expectancy gap between the most and less frail individuals is close to 17 years in 2000 and increases to 25 years by 2100. Panel 1(b) shows the positive correlation between life expectancy and ability, although the difference in life expectancy by ability group is smaller than in Panel 1(a). This is due to the fact that each ability group is comprised of individuals with different mortality frailties. The weights at birth are given in Table 1. Nevertheless, one should be aware of the fact that these weights will differ as individuals age. In particular, the proportion of less frail individuals will increase, the fewer surviving individuals are left in each birth cohort. Panel 1(c) shows the evolution of the total fertility rate, which is assumed to be the same for all nine population groups. This assumption although restrictive will allow us to focus on the mortality channel. Thus, we prevent that our results will be affected by the fact that each group face a different household size (i.e. total childrearing cost).[4] Panel 1(d) shows the population distribution in years 2015, 2050, and 2100 that results from the fertility and mortality depicted in Panels 1(a)-1(c).

[Figure 1 about here.]

4.2 The economy

In this subsection we first explain the main parametric components of the US pension system (our baseline) and of the counterfactual experiment that is implemented in our simulations. Second, we briefly introduce the main model economy parameters.

[4] Indeed, differences in mortality rates below age 15 among the different groups are almost negligible.

Parametric components of the pension system. The US pension system started in year 1935. The main feature of the US pension system compared to many other pension systems of OECD countries is that its replacement rate is progressive.

Given a retirement age J_R, the benefits claimed at retirement can be modeled with the following information: i) current contribution weights, ρ_j; ii) the capitalization index, I_t; iii) pensionable earnings, p; iv) the replacement rate, $\psi(p)$; and v) the retirement incentives or penalties, $\lambda(J_R)$. Table 2 shows the parametrization of three of the main components of the US pension system (OASDI). Note that the pensionable earnings are derived by the accumulation of labor income histories, which are endogenously determined in the model. The value of $\rho_j = \rho$ for all j is chosen so as to obtain a pension cost to output ratio in year 2013 of 5%. This is done with a value of ρ of 1/35 or, equivalently, the average of 35 years of work. Not surprisingly, this value coincides with the US pension system that takes into account the highest 35 labor income along the working life of an individual. In our case, the highest labor incomes are close to retirement, hence we assume that $\rho = 0$ is initially zero and it is 1/35 during the last 35 years at work.

[Table 2 about here.]

As Table 2 shows the maximum replacement rate is 0.90, if an individual's pensionable earnings is less than six times the average labor income of the economy, and the replacement rate tends to zero the larger the pensionable earnings are.[5] Let $\bar{y}_t$ be the average labor income of the economy in year t, which is calculated as

$$\bar{y}_t = \frac{\text{Total labor income in year } t}{\text{Total number of workers in year } t}.$$

As the second column in Table 2 shows the retirement incentives and penalties depend on the difference between the actual retirement age and the normal retirement age. In our simulations, we assume a fix retirement age of 65 (i.e. $J_R = 65$). When the US pension system was introduced the normal retirement age (denoted by J_N) was set at 65 years. However, recent reforms have established a gradual increase in the normal retirement age that depends on the year of birth of the retiree. The evolution of the normal retirement

[5] In our simulations, the maximum pensionable earnings of high ability individuals is almost twice the minimum pensionable earnings of low ability individuals. Thereby, all simulated incomes are below the maximum pensionable earnings.

age can be seen in the fourth column of Table 3.[6] Additional incentives and penalties were also introduced to guarantee the sustainability of the pension system. In particular, individuals who decide to claim their pension benefits before the normal retirement age are subject to penalties, while those who decide to retire after the normal retirement age receive credits. Table 3, column 2, shows the yearly credits received by delaying the age of retirement from J_N to age 70, while column 3 shows the yearly penalties from claiming the retirement benefits before the normal retirement age.

[Table 3 about here.]

The simulation results shown in Section 5 are based on the comparison of two alternative pension systems. First, one pension system whose pension formula resembles the largest 'retirement benefit program' of the US pension system (OASDI) – see the parametric components in the first row of Table 2. Second, another pension system with a fix replacement rate. For comparative purposes between the baseline and the counterfactual, we set the fix replacement rate at 0.385, which is the long run replacement rate value that we have obtained for the average learning ability individual in the baseline simulation.

Model economy parameters. For comparability with the existing literature we opted for using the same parameter values of the closest model to ours (Ludwig et al., 2012), which were also calculated for the US economy. Table 4 reports the main model economy parameters.

[Table 4 about here.]

We run the model assuming a standard CIES utility function

$$U(c_{t,j}, z_{t,j}) = \frac{1}{1-1/\sigma}\left[\left(c_{t,j}^{\phi} z_{t,j}^{1-\phi}\right)^{1-1/\sigma} - 1\right],$$

which, in the interior solution, will imply the following Frisch elasticities on education and labor supply

$$\frac{R_t^H}{e_{t,j}}\frac{\partial e_{t,j}}{\partial R_t^H} = \frac{1}{1-\gamma},$$
$$\frac{R_t^H}{\ell_{t,j}}\frac{\partial \ell_{t,j}}{\partial R_t^H} = \frac{1-\ell_{t,j}-e_{t,j}}{\ell_{t,j}}(\sigma+\phi(1-\sigma)) + \frac{1}{1-\gamma}\frac{e_{t,j}}{\ell_{t,j}},$$

[6] This information can be checked at www.socialsecurity.gov

respectively. Moreover, the intertemporal elasticity of substitution on consumption is $\frac{\sigma}{\sigma+\phi(1-\sigma)}$. As a consequence, while the elasticity of education and consumption are constant and equal to 2.86 and 0.71, respectively, the Frisch elasticity of labor supply changes over the working life, across cohorts, and by ability level. For instance, for cohorts born after 2000 the Frisch elasticity of labor supply at ages 30, 40, and 50 will be close to 2.4, 1.2., and 0.9, respectively.

5 The Simulations: Progressive replacement vs. flat replacement

In this section we show the main simulation results, which we divide in three parts. In the first part, we analyze the impact of the two pension systems on the decision making process (consumption, leisure, educational investment, and labor supply) and its consequences for the net labor income, pension benefits, and the accumulation of physical and human capital. The baseline simulation considers a progressive replacement rate that mimics the OASDI pension system in the US. The alternative simulation is based on a constant replacement rate of 0.385, that corresponds to the long-run replacement rate for the average individual in the baseline simulation. In the second part, we study how each pension system redistributes the resources among individuals with different learning ability levels. We pay a special attention on the difference between the social contributions paid and the pension benefits received by each ability group. This is because the government is considered to distinguish individuals by their ability group but not to know about the frailty group. Finally, we compare the effect of each pension system on economic growth.

5.1 Behavioral effects

In Figures 2-4 we begin by describing the optimal decisions for the cohort born in 2000. It is assumed that individuals born in year 2000 have a (cohort) life expectancy at birth of 85, 88.5, and 92 years for those with low ability, average ability, and high ability, respectively. While the life expectancy at age 15 is 71, 74, and 77 for those belonging to the low, average, and high ability groups, respectively. Moreover, this cohort is expected to raise, on average, one child (i.e. two children per couple) and the mean-age of childbearing is assumed to be 28 years old, which implies that the household size (measured in units of equivalent adult consumers) peaks when the household head is of age 38.

[Figure 2 about here.]

Figure 2 shows how individuals born in year 2000 optimally allocate their time among leisure, labor supply, and educational investment over their life cycles by their ability level. On average this cohort divides of its remaining lifetime at age 15 between 22.6 (high) and 24.7% (low) for work and between 11.5 (low) and 13.2% (high) for educational investment. The lower labor supply over the life cycle of the high ability group relative to that of the low ability group is due to the longer remaining life expectancy of the former group. Indeed, although individuals with high ability enter four years later in the labor market –from 18 years (low) to 22 years (high)–, they also supply more intensive labor while at work. As a result, differences in total hours worked between the three ability groups, weighted by the corresponding survival probabilities, are almost negligible. The time spent on education is clearly higher for those with higher ability than for those with lower ability. For a given interest rate, this is because individuals with higher ability have a higher marginal benefit of an additional hour of education than those with a lower ability.

We can see in Figure 2 that differences in time use between the flat replacement rate and the progressive replacement rate are small. Comparing the dotted line with the solid line notice that a flat replacement rate marginally reduces the time for leisure for all ability groups. Early in life, the labor supply increases for individuals with a lower ability, while it decreases for average and high ability individuals. Late in the working life, however, the labor supply marginally increases for the average and high ability groups. As a consequence, a flat replacement rate has a positive effect on the marginal benefit of education for the average and high ability individuals but it is negative for the low ability group.

[Figure 3 about here.]

Household consumptions, cash-in-hands, and saving over the life cycle by ability level for the cohort born in year 2000 are shown in Figure 3. The household consumption profiles, which are comprised of the adult consumption plus the consumption of dependent children, are shown in the upper panels of Figure 3. Cash-in-hand profiles for this cohort are shown in the middle panels. Cash-in-hand includes net labor income, pension benefits, asset income, and (capitalized) bequest received. Despite the inclusion of pension benefits and the capitalized bequest, consumption over the life cycle is mainly financed by the labor income. Table 5 shows that the present value of all transfers received –bequest and pension benefits– and paid –social contributions– are close to cancel each other and never exceeds 2% of the present value of lifetime labor income.

[Table 5 about here.]

Saving profiles for the cohort born in year 2000 by ability group are shown in the bottom panels of Figure 3. Individuals borrow from age 15 to 30 regardless their ability level. After age 30, the cash-in-hand exceeds consumption until mid-80s. Thus, even thought individuals mandatorily retire at age 65 and they do not save with a bequest motive in mind, this cohort still saves during the first twenty years in retirement.

[Figure 4 about here.]

Figure 4 shows the evolution of our three endogenous stock variables for each ability group: assets, pension earnings, and human capital stock. As the upper panels in Figure 4 show, individuals with higher ability accumulate more assets. This result is consistent with the positive correlation between the ability level and the life expectancy. Pension earnings are also positively related to the ability level since individuals with higher ability also have a higher labor income (see the middle panels). The average replacement rate for this cohort is 0.42, 0.38, and 0.34 for the low, average, and high ability groups in the baseline simulation, while the replacement rate in the flat replacement scenario was set at 0.385 for all ability groups. The next subsection explains in more detail the redistributive consequences of implementing each replacement rate. Due to the longer and higher intensive educational investment of the higher ability groups, the human capital stock is also higher for those individuals with higher ability. Comparing the dotted curves with the solid curves, we can observe that the implementation of a flat replacement rate also boosts the accumulation of human capital for individuals above the low ability level, while it reduces the optimal accumulation of human capital for individuals with low ability. As explained in Section 2.3.1 –see Eq. 21–, this is because a higher (lower) replacement rate reduces (increases) the effective social security tax rate, which induces a higher (lower) investment in education. At the macro level and assuming a close economy, the higher human capital stock leads to an increase in the return to physical capital and further accumulation of physical capital. As a result, for a given social contribution rate, we can conclude that a higher replacement rate raises the accumulation of physical and human capital.

5.2 Redistributive effects

This subsection focuses on the redistributive consequences of implementing a progressive replacement rate versus a flat replacement rate. Using our simulations we are interested in knowing who are the net beneficiaries of each pension system. Assuming that the government knows the ability of each individual but not the life expectancy, we start looking at how each simulation scenario redistributes resources among the three ability groups from an individual perspective (cohort) and from a group perspective (cross-sectional).

[Figure 5 about here.]

The goal of the US pension system is to redistribute resources from high income individuals to low income individuals through the progressive replacement rate formula. To study the redistribution of resources, we first calculate the total benefits received minus contributions paid by each ability group in a given year

$$\sum_{\mu=1}^{3}\sum_{j=66}^{120} b_{t,j}(\theta,\mu)N_{t,j}(\theta,\mu) - \tau_t \sum_{\mu=1}^{3}\sum_{j=15}^{65} y_{t,j}(\theta,\mu)N_{t,j}(\theta,\mu).$$

In our general equilibrium model with perfect foresighted individuals, Figure 5(a) shows how the US replacement rate formula (baseline) does not redistribute resources among our three ability groups. In contrast, Figure 5(b) shows how a constant replacement rate of 0.385 implies that the low and average ability groups transfer each year around 4% of the total pension budget to the high ability group. As Figure 6(b) shows this is because the relative pension cost of individuals with high ability group to the output produced by the same ability group is the highest in the flat replacement rate scenario, while this cost is lower for the low ability group.

[Figure 6 about here.]

According to Figure 6 the total pension to output ratio will progressively increase from five percent at the beginning of the twenty first century to over twelve percent in year 2100, even after implementing the latest reforms of the OASDI pension system.

However, neither Figure 5 nor Figure 6 are informative at the individual level. To understand the redistributive goal of the pension system, we further calculate for each individual type the ratio between the present value of the benefits received and social contributions paid over the life cycle (or social security wealth) and the present value of the stream of labor income as follows

$$\int_{\boldsymbol{\mu}} \frac{\sum_{j=66}^{120} \frac{b_{c+j,j}(\theta,\mu)}{\prod_{z=15}^{j} D_z} - \sum_{j=15}^{65} \frac{\tau_{c+j} y_{c+j,j}(\theta,\mu)}{\prod_{z=15}^{j} D_z}}{\sum_{j=15}^{65} \frac{y_{c+j,j}(\theta,\mu)}{\prod_{z=15}^{j} D_z}} dG(\theta,\mu),$$

where D_z is the discount rate in year z. Since the social security wealth value depends to a great extent on the discounting factor used, in Figure 7 we show the ratio between the social security wealth and the present value of labor income by ability level under two different discount factors. The upper panels use as discounting factor the (real) interest rate of the economy. Note that since there is no annuity market, the values obtained in the upper panels coincide with those used by our economic agents to make their optimal decisions.[7] The bottom panels show the ratio using the survival probability as the only discount factor. Results presented in Figure 7 lead to the following four conclusions: i) The social security wealth as a fraction of the present value of labor income is very similar across the different ability groups and for the different discount factor in the baseline simulation; ii) given that the present value of labor income increases with the ability level, the social security wealth is lower (higher) for the highest ability group when the discount factor is the interest rate (survival probability); iii) with a flat replacement level, if we use the interest rate as a discount factor, the social security wealth relative to the present value of labor income is smaller, the lower the ability level is; and iv) if we only consider the survival probability as the discount factor, in a pension system with a flat replacement rate individuals with higher abilities receive on average more benefits and contributions than individuals with lower ability.

[Figure 7 about here.]

5.3 Macroeconomic effects

As it has been shown in Figure 6 the population aging process will raise the future cost of the OASDI pension system. Nevertheless, the increase in life expectancy and the decline in fertility does not necessarily imply a burden (Lee et al., 2014). To understand the macroeconomic consequences of each

[7]Simultaneously discounting the net social security transfers and labor income by the interest rate and the survival probability, the results will be qualitatively similar to those shown in the upper panels of Figure 5.

replacement rate, we decompose the output per capita (Y/N) in the following four components:

$$\frac{Y_t}{N_t} = A_t \left(\frac{K_t}{Y_t}\right)^{\frac{\alpha}{1-\alpha}} \frac{H_t}{N_{t,15+}} \frac{1}{1 + \frac{ydr_t}{1+oadr_t}}. \tag{30}$$

The first term on the right-hand side is the exogenous productivity growth A_t, followed by the capital to output ratio (K_t/Y_t), the average human capital employed per individual older than 15 years old $H_t/N_{t,15+}$, and a ratio of demographic dependency ratios. The ydr_t stands for the youth dependency ratio in year t and $oadr_t$ is the old-age dependency ratio in year t. Hence, by definition only the second and third components are affected by changes in the replacement rate of the pension system. Moreover, Eq. (30) clearly shows that population aging might have a positive impact on income per capita.

Figure 8 shows the evolution of the youth and old-age dependency rates for each ability group. According to our simulated demography, the youth dependency rate stabilizes around 3 children per 10 people within ages 15-65 from year 2010, while the number of elderly people will increase from 2 in year 2000 up to a maximum of 6.5 per 10 people within ages 15-65 in year 2100. Notice that the old-age dependency rate increases faster for the high ability group because this group experiences a longer life expectancy. Given our fixed retirement age and an increasing educational attainment, our demographics implies that the total number of non-workers will exceed the total number of workers by the end of the twenty first century.

[Figure 8 about here.]

Our agents will react to the demographic changes by modifying their consumption pattern as well as their supply of productive factors: hours worked, human capital stock, and assets. For convenience, we rewrite the average human capital employed by ability level as

$$\frac{H_t}{N_{t,15+}} = \sum_{\theta=1}^{3} h_t(\theta)\ell_t(\theta)\frac{N_{t,15+}(\theta)}{N_{t,15+}}, \tag{31}$$

where

$$h_t(\theta) = \frac{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} h_{t,j}(\theta, \mu, P_{t-j}) \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}, \tag{32}$$

(Average human capital per worker of type θ)

$$\ell_t(\theta) = \frac{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} \ell_{t,j}(\theta, \mu, P_{t-j}) N_{t,j}(\theta, \mu)}{\sum_{j=15}^{\Omega} \sum_{\mu=1}^{3} N_{t,j}(\theta, \mu)}. \tag{33}$$

(Average time devoted to work per adult of type θ)

The macroeconomic consequences of the behavioral reactions are summarized in Figure 9. First, given a mandatory retirement age, the time spent working over the life cycle will decline on average from 28% to 23% due to the longer life expectancy –see the top panels in Figure 9–. For the same reason, individuals with higher ability will experience a faster decline than those with a lower ability. Notice that this result holds even with a flat replacement. Second, the decline in the intensive labor supply over the life cycle is offset with an increase in the average human capital employed from three units in 2000 to over four units in 2100 –see the middle panels in Figure 9–. Note that the increase is even higher when a flat replacement rate is implemented due to the higher pension benefits received by individuals with an average and high ability. Third, on average capital to output ratio will increase from 2.5 in 2000 to 2.8 in 2100. The evolution of this variable is however quite different among the three ability levels and replacement rates. In particular, in our baseline simulation, individuals with low ability have the highest capital-to-output ratio at the beginning of the twenty first century due to the fact that they do not borrow as much as the high ability group early in life. This process is however reversed at the end of the twenty first century because the mean-age of asset holders increase. Under a flat replacement rate, the capital-to-output ratio between the two ability groups do not cross over because the low ability group needs additional savings to finance their retirement, whereas the high ability group compensates the higher benefits with lower savings (i.e. crowding-out effect).

[Figure 9 about here.]

The small increase in the capital-to-output ratio is translated into lower (real) interest rates along the twenty first century as shown in Figure 10. The bump between 2030 and 2060 is the result of the progressive retirement of the baby-bust, who work more hours at the end of their working lives. It is also interesting to point out that the higher interest rate in the flat replacement rate simulation relative to the baseline is the consequence of the additional incentives introduced by the pension system on the accumulation of human capital.

[Figure 10 about here.]

Finally, combining the results plotted in Figures 8-10 through Eq. (30), we derive the growth of the income per capita by ability level. Figure 11 shows that the income per capita increases in both scenarios for all ability levels mainly due to the accumulation of human capital. Note that the values plotted in Figure 11 do not include the exogenous annual productivity growth of 2%. However, the income per capita growth differs by ability level. For instance, the average income per capita growth for individuals with low ability is less than 10% in one hundred years, while the increase is above 25% for individuals with high ability. As a consequence, regardless the pension system implemented, the model predicts a progressive increase in inequality between those individuals with low, average, and high ability. This result stems from the fact that the average capital-to-output ratio of low ability individuals declines along the twenty first century.

[Figure 11 about here.]

6 Conclusions

This paper builds a computable overlapping generation model of labor supply, human capital formation, and physical capital accumulation with heterogeneity in human capital and life expectancy. The model analyzes the economic consequences of running a pension system with a progressive replacement rate versus a flat replacement rate in the context of within cohort heterogeneity in life expectancy and human capital.

We apply the model to the US economy through the implementation of the OASDI pension system. The overall demographic features of the US population are replicated in our model and we further introduce heterogeneity within cohort by life expectancy and learning ability. The within cohort structure is set so as to obtain a positive correlation between the ability level and frailty (Lleras-Muney, 2005). Consequently, in our model, agents with longer schooling also have an average longer life expectancy.

Our simulations suggest the following results. First, the OASDI pension system does not redistribute resources from low ability individuals to high ability individuals. Second, we obtain that future income per capita will increase for all ability levels due to the further accumulation of human capital. However, due to the positive correlation between the ability and life expectancy, the inequality between the different ability groups will increase during the twenty first century. Third, the model shows that there exists a positive link between the generosity of the pension system and the accumulation of human capital.

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7 Appendix: Household problem

We assume household heads start making decision when they complete the elementary school (8th grade) at the age of 15. They choose the household consumption path (c), the additional education (e) they still want to acquire, and the leisure time (z). The remaining available time is devoted to work.

The expected utility V of a household head that belongs to group θ at age x depends on the assets holding a, the average pension earnings p, the stock of human capital h, and the demographic states η (household size measured in units of equivalent adult consumption). For simplicity in the exposition we get rid of the time variable and denote the next period with the symbol '$'$'. Our agent solves the following problem:

For $j \leq J_R$:

$$V(a,p,h,\eta) = \max_{c,z,e} \{U(c,z) + \beta\pi' V(a',p',h',\eta')\} \tag{34a}$$

subject to

$$a' = Ra + (1-\tau)whA(1-z-e) - c\eta, \tag{34b}$$
$$p' = Ip + \rho wAh(1-z-e), \tag{34c}$$
$$h' = h(1-\delta) + q(h,e), \tag{34d}$$
$$\lambda_1(1-z-e) = 0, \tag{34e}$$
$$\lambda_2 z = 0, \tag{34f}$$
$$\lambda_3 e = 0, \tag{34g}$$

For $j > J_R$:

$$V(a,p,h,\eta) = \max_{c} \{U(c,1) + \beta\pi' V(a',p',h',\eta')\} \tag{34h}$$

subject to

$$a' = Ra + \psi(p)p - c\eta, \tag{34i}$$
$$p' = p, \tag{34j}$$
$$h' = h(1-\delta), \tag{34k}$$

and the boundary conditions

$$a_0 = p_0 = 0, h_0 = 1, \text{ and } a_{\omega+1}, p_{\omega+1}, h_{\omega+1} \geq 0, \tag{34l}$$

where c is the household consumption, e is the education effort, ℓ is the intensive labor supply, β is the subjective discount factor, π_x is the conditional probability of surviving to age x, R is the capitalization factor, τ is the social security contribution rate, w is the wage rate per unit of human capital, δ is the human capital depreciation rate, and $q(h, e)$ is the human capital production function (with $q_h, q_e > 0$ and $q_{hh}, q_{ee}, q_{he} < 0$).

7.1 First-order conditions and envelope conditions

Let define Ξ and Σ as the marginal rate of substitution between pension wealth and assets and human capital and assets, respectively. The first-order conditions and envelope conditions are

For $j \leq J_R$:

$$U_c = \beta\pi' V_{a'}\eta, \tag{35}$$
$$U_z = \frac{U_c}{\eta} whA(1-\tau^L) + \lambda_1 - \lambda_2, \tag{36}$$
$$0 = \frac{U_c}{\eta}[whA(1-\tau^L) - \Sigma' q_e(h,e)] + \lambda_1 - \lambda_3, \tag{37}$$

and

$$V_a = R\beta\pi' V_{a'}, \tag{38}$$
$$V_p = I\beta\pi' V_{p'}, \tag{39}$$
$$V_h = \beta\pi' V_{a'}\left[wA(1-z-e)(1-\tau^L) + \Sigma' R^h\right], \tag{40}$$

where τ^L is the effective social security tax on labor

$$\tau^L = \tau - \rho\Xi' \quad \text{(effective social security tax on labor)}, \tag{41}$$
$$R^h = 1 + q_h(h,e) - \delta \quad \text{(capitalized return to education)}. \tag{42}$$

Notice that the law of motion of the marginal rate of substitutions for the state variables are:

$$\Xi = \Xi' \frac{I}{R}, \tag{43}$$
$$\Sigma = \Sigma' \frac{R^h}{R} + \frac{wA(1-z-e)(1-\tau^L)}{R}. \tag{44}$$

Recall that when the marginal rate of substitution is greater than one, the household head will have an incentive to transfer resources from assets to another state variable.

For $j > J_R$:

$$U_c = \beta \pi' V_{a'} \eta, \tag{45}$$

and

$$V_a = R\beta\pi' V_{a'}, \tag{46}$$
$$V_p = \beta\pi' V_{a'} \left(\psi(p) + \psi'(p)p + \Xi'\right), \tag{47}$$
$$V_h = \beta\pi' V_{h'}(1-\delta). \tag{48}$$

Since in the last period $V_{h'} = 0$, we have the following marginal rates of substitutions:

$$\Xi = \Xi' \frac{1}{R} + \frac{\psi(p) + \psi'(p)p}{R}, \tag{49}$$
$$\Sigma = 0. \tag{50}$$

Solutions:

Assuming the following instantaneous utility functional form:

$$U(c, z) = \frac{\left(c^{\phi} z^{1-\phi}\right)^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}.$$

Using the foc and envelope conditions, we have the following cases

For $j > J_R$:

$$\tilde{V}_a = R\beta\pi'\tilde{V}_{a'}\left[\frac{A'}{A}\right]^{\phi(1-\frac{1}{\sigma})-1}, \tag{51}$$

$$\tilde{U}_c = \tilde{V}_a\frac{\eta}{R}, \tag{52}$$

$$\tilde{c} = \left[\frac{\tilde{U}_c}{\phi}\right]^{\frac{1}{\phi(1-\frac{1}{\sigma})-1}} \tag{53}$$

$$\Xi = \Xi'\frac{1}{R} + \frac{\psi(p)+\psi'(p)p}{R}, \tag{54}$$

$$\Sigma = 0. \tag{55}$$

For $j \le J_R$:

If $\lambda_1 = \lambda_2 = \lambda_3 = 0$ (interior solution), we have

$$\tilde{U}_c = \beta\pi'\tilde{V}_{a'}\eta\left[\frac{A'}{A}\right]^{\phi(1-\frac{1}{\sigma})-1}, \quad \frac{\tilde{U}_z}{\tilde{U}_c} = \frac{wh(1-\tau^L)}{\eta}, \quad q_e(h,e) = \frac{A}{A'}\frac{wh(1-\tau^L)}{\tilde{\Sigma}'}, \tag{56}$$

which implies the following optimal control variables

$$h = \frac{1}{1-\delta}\left[h' - \varphi\left(\frac{A'}{A}\frac{\varphi\gamma\tilde{\Sigma}'}{w(1-\tau^L)}\right)^{\frac{\gamma}{1-\gamma}}\right],$$

$$e = \frac{1}{h}\left(\frac{A'}{A}\frac{\varphi\gamma\tilde{\Sigma}'}{w(1-\tau^L)}\right)^{\frac{1}{1-\gamma}},$$

$$\tilde{\text{clr}} = \frac{\phi}{1-\phi}\frac{wh(1-\tau^L)}{\eta},$$

$$z = \left[\frac{\phi}{\tilde{U}_c}\right]^{\sigma}\tilde{\text{clr}}^{\phi(1-\sigma)-\sigma},$$

$$\tilde{c} = z\tilde{\text{clr}},$$

$$\Xi = \Xi'\frac{I}{R},$$

$$\tilde{\Sigma} = \tilde{\Sigma}'\frac{A'}{A}\frac{R^h}{R} + \frac{w(1-z-e)(1-\tau^L)}{R}.$$

Notice the utility function and the human capital production function satisfies the Inada conditions. Hence, λ_2 and λ_3 are always zero.

If $\lambda_1 > 0$ (Corner solution), we have

$$\tilde{U}_c = \beta\pi'\tilde{V}_{a'}\eta, \qquad \frac{\tilde{U}_z}{\tilde{U}_c} = \frac{A'}{A}\frac{\tilde{\Sigma}' q_e(h,e)}{\eta}, \qquad z = 1 - e, \tag{57}$$

which implies the following optimal control variables

$$\underbrace{\log\left[1 - \frac{(1-\delta)\varepsilon^{\star\frac{1}{\gamma}}}{h' - \varphi\varepsilon^\star}\right] + [\sigma - \phi(\sigma-1)]\log\left[\frac{h' - \varphi\varepsilon^\star}{1-\delta}\varepsilon^{\star 1-\frac{1}{\gamma}}\right]}_{\text{LHS}}$$
$$= \underbrace{\sigma\log\left[\frac{\phi}{\tilde{U}_c}\right] - [\sigma - \phi(\sigma-1)]\log\left[\frac{1-\phi}{\phi}\frac{A'}{A}\frac{\tilde{\Sigma}'}{\eta}\gamma\varphi\right]}_{\text{RHS}}, \tag{58}$$

$$\begin{aligned}
\varepsilon_{i+1} &= \varepsilon_i + \xi(LHS - RHS),\\
h &= \frac{h' - \varphi\varepsilon}{1-\delta},\\
e &= \frac{(1-\delta)\varepsilon^{\frac{1}{\gamma}}}{h' - \varphi\varepsilon},\\
z &= 1 - e,\\
\tilde{\text{clr}} &= \frac{A'}{A}\frac{\phi}{1-\phi}\frac{\tilde{\Sigma}' q_e}{\eta},\\
c &= z\text{clr}.
\end{aligned}$$

For convenience we de-trend all the necessary variables by the productivity:

$$\tilde{c} = c/A, \tag{59}$$
$$\tilde{U}_c = U_c / A^{\phi(1-\sigma)-1}, \tag{60}$$

A priori $z > 0$, or $\lambda_2 = 0$, because of the Inada conditions. Thus, the following four alternatives are possible (in order of programming)

Case 0 (Interior solution): $\lambda_1 = \lambda_3 = 0$, where $\overline{lcr}_x = \frac{1-\phi}{\phi} \frac{\eta_x}{\mu_x} \frac{1}{w_x}$

$$\begin{cases} \text{If } h^* = \frac{1}{1-\delta^h} \left[h_{x+1} - \varphi \left(\frac{\varphi \gamma \xi^H_{x+1}}{w_x A_x (1-\tau^L_x)} \right)^{\frac{\gamma}{1-\gamma}} \right] < \bar{h} \text{ then } h_x = h^*, \\ \text{otherwise } h_x = \frac{1}{1-\delta^h} \left[h_{x+1} - \varphi \left(\frac{\varphi \gamma \xi^H_{x+1}}{w_x A_x (1-\tau_x)} \right)^{\frac{\gamma}{1-\gamma}} \right], \end{cases} \tag{61}$$

The above piecewise function must be programmed in that order because there exists a positive relationship between current stock of human capital and the marginal rate of substitution between pension wealth and assets, i.e. $\frac{\partial h_x}{\partial \xi^P_{x+1}} > 0$.

$$\tau^L_x = \tau_x - \xi^P_{x+1} \begin{cases} 1/J_R & \text{if } h_x < \bar{h}, \\ 0 & \text{otherwise.} \end{cases} \tag{62}$$

$$e_x = \frac{1}{h_x} \left[\frac{h_{x+1} - h_x(1-\delta^h)}{\varphi} \right]^{\frac{1}{\gamma}}, \tag{63}$$

$$\frac{\tilde{c}_x}{\eta_x} = \left[\frac{\phi}{\eta_x \tilde{U}_c} \right]^{\frac{1}{\sigma}} \left[\frac{\overline{lcr}_x}{h_x(1-\tau^L_x)} \right]^{(1-\phi)\left(\frac{1}{\sigma}-1\right)}, \tag{64}$$

$$z_x = \mu_x \frac{\tilde{c}_x}{\eta_x} \frac{\overline{lcr}_x}{h_x(1-\tau^L_x)} \tag{65}$$

Case I: If $e = 0$ (when $\xi^H_{x+1} = 0$) and $z < 1$ or $(\lambda_1 = 0, \lambda_3 > 0) \Rightarrow$ We do not need to change the equations

Case II: If $e = 0$ (when $\xi^H_{x+1} = 0$) and $z > 1$ or $(\lambda_1 > 0, \lambda_3 > 0)$

$$h_x = \text{idem} \tag{66}$$

$$e_x = \text{idem}, \tag{67}$$

$$\frac{\tilde{c}_x}{\eta_x} = \left(\frac{\eta_x \tilde{U}_c}{\phi} \right)^{\frac{1}{\phi(1-\sigma)-1}}, \tag{68}$$

$$z_x = 1 \tag{69}$$

Case III: If $e > 0$ (when $\xi^H_{x+1} > 0$) and $z + e > 1$ or $(\lambda_1 > 0, \lambda_3 = 0)$. Find the value of h that satisfies:

$$e(h) = \frac{1}{h}\left(\frac{h_{x+1} - h(1-\delta^h)}{\varphi}\right)^{\frac{1}{\gamma}}, \tag{70}$$

$$\log \frac{\eta_x \tilde{U}_c}{\phi} = \Xi_0 \log \frac{\xi^H_{x+1} q_e(h, e_x(h))}{w_x A_x \overline{lcr}_x} + \Xi_1 \log \frac{1 - e(h)}{\mu_x}, \tag{71}$$

$$\frac{\tilde{c}_x}{\eta_x} = \frac{1 - e_x}{\mu_x} \frac{\xi^H_{x+1} q_e(h_x, e_x)}{w_x A_x \overline{lcr}_x}, \tag{72}$$

$$z_x = 1 - e_x, \tag{73}$$

where

$$\Xi_0 = \phi(1-\sigma) - 1, \qquad \Xi_1 = -\sigma \tag{74}$$

Computational strategy: First, we create a grid of feasible final consumption values. Second, for $i = 1$ and given a final value of the human capital $h_\Omega(i) = 1$ we recursively seek for the final value of $p_\Omega(i)$ that gives a $p_0(i) = 0$ when $h_0(i) = 1$. Third, we store the triplet, including the value of the initial assets, associated to the previous case $\{p_0(i) = 0; h_0(i) = 1; a_0(i)\}$. Fourth, we seek for the final human capital value $h_\Omega(i)$ that makes $a_0(i) = 0$ using the following algorithm:

```
i=1
Err^i=1
while Err^i>0.001
i=i+1
Err^i=abs(h_\Omega(i-1)-0)
If Err^i-Err^{I-1}>0
\begin{cases}
\xi(i)=\xi(i-1)
h_\Omega(i)=h_\Omega(i-1)+\xi(i)(a_\Omega(i-1)-0)
else
\xi(i)=0.90\xi(i-1)
h_\Omega(i)=h_\Omega(i-1)+\xi(i)(a_\Omega(i-1)-0)
end
end
```

8 Lifespan heterogeneity

To complete a CGE model, we found that it is necessary to have a continuum of life expectancies within cohorts. In what follows I suggest a demographic model that accounts for such population. The easiest and most coherent way I can think about is using the temporal component of the Lee-Carter model as follows:

Let the temporal component of mortality gains at time t be

$$k_{t+1} = \mu(\theta) + \phi k_t, \tag{75}$$

where the drift is

$$\mu(\theta) = \bar{\mu} + \theta \text{ with } \theta \sim \mathcal{U}(-\alpha, \alpha) \text{ and } 0 < \alpha < \bar{\mu}. \tag{76}$$

Thus, $\bar{\mu}$ would match the observed drift in the population. For simplicity, each population group would have a different θ value, and hence a different life expectancy. An additional and convenient assumption is that there is no possibility of moving from one θ value to another. This model can be applied from year 1912 onwards to the US population, as in the previous simulation. However, several issues still arise. For example, if we apply this mortality model from age 0, shall the fertility pattern be the same for all population groups? The main problem is that we need to find a balance between realism and clean results that we know they are not contaminated by other effects.

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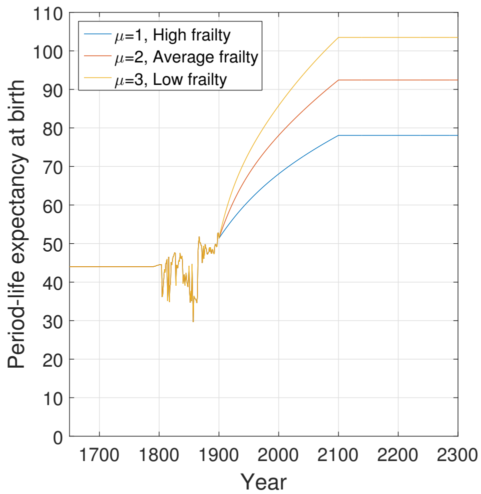
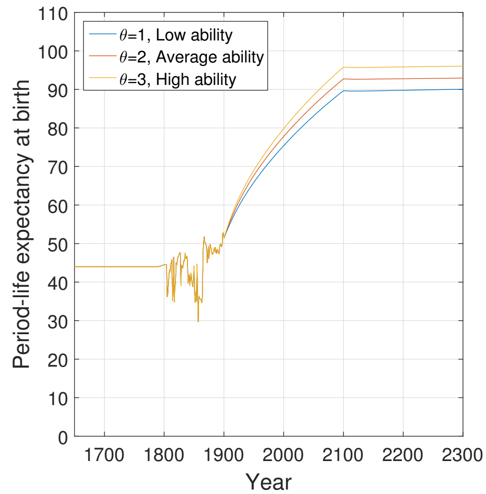
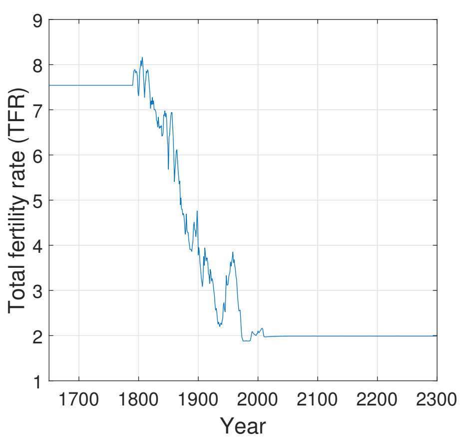


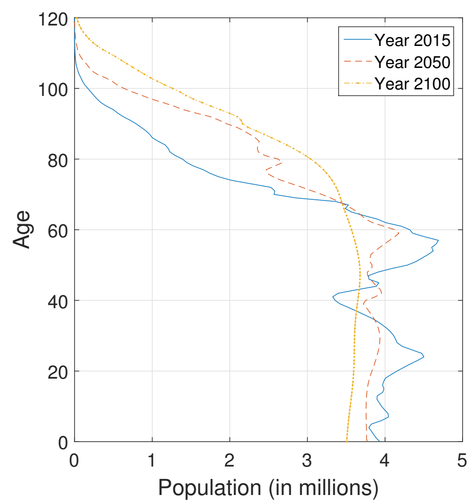
(a) Life expectancy by frailty (μ)



(b) Life expectancy by ability (θ)

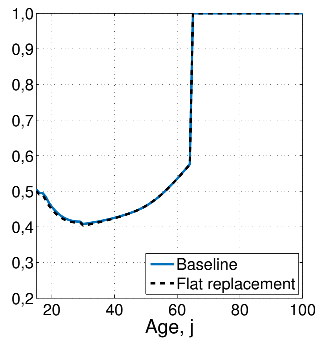


(c) Total fertility rate

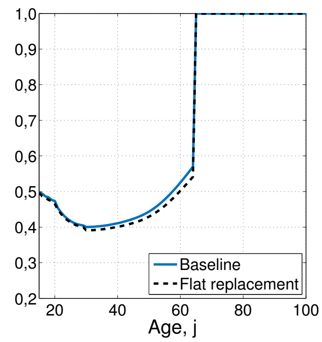


(d) Total population distribution

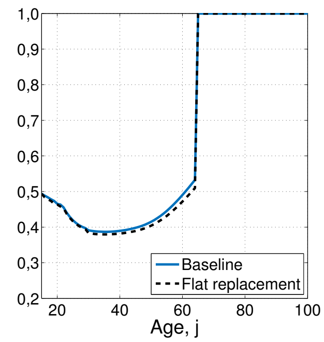
Figure 1: US demographics



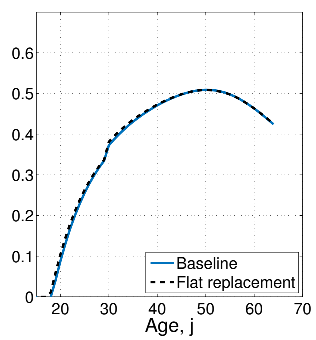
(a) Leisure time, low ability



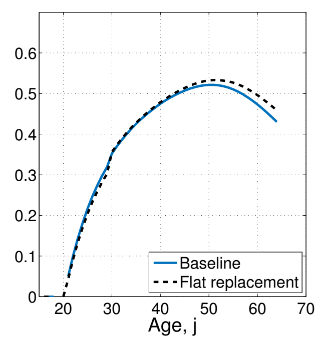
(b) Leisure time, avg. ability



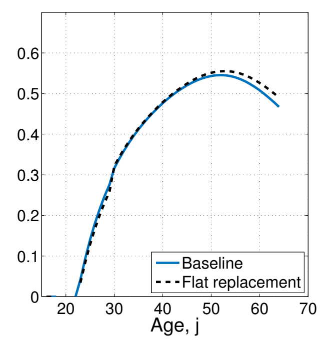
(c) Leisure time, high ability



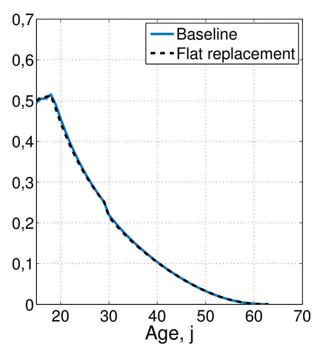
(d) Labor supply, low ability



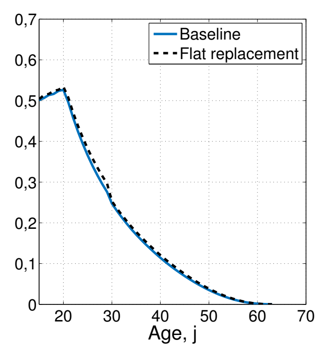
(e) Labor supply, avg. ability



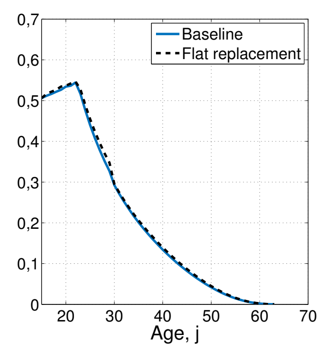
(f) Labor supply, high ability



(g) Educational investment, low ability

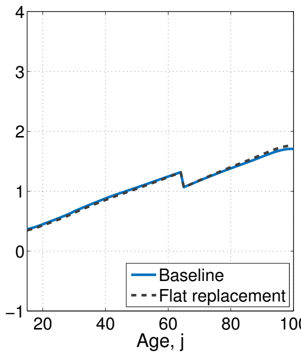


(h) Educational investment, avg. ability

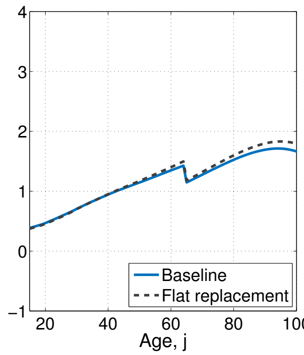


(i) Educational investment, high ability

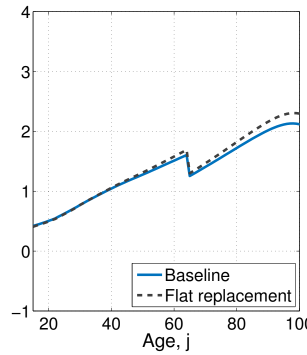
Figure 2: Time use over the life cycle by ability level: 2000 birth cohort



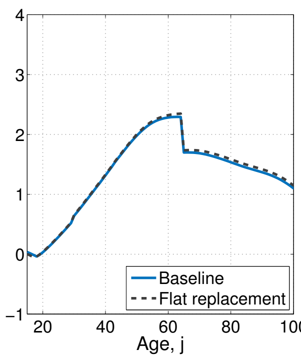
(a) Household consumption, low ability



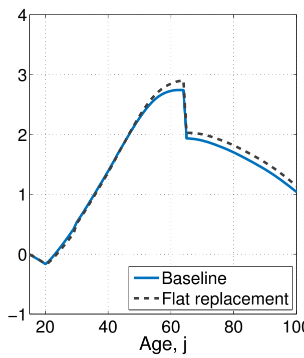
(b) Household consumption, avg. ability



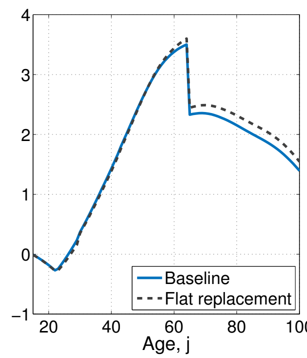
(c) Household consumption, high ability



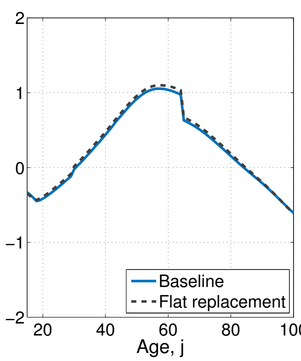
(d) Cash-in-hand, low ability



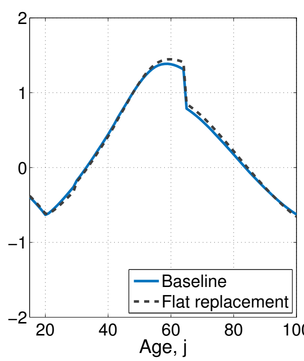
(e) Cash-in-hand, avg. ability



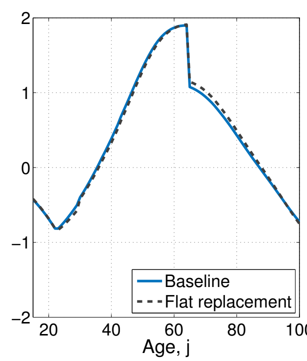
(f) Cash-in-hand, high ability



(g) Saving, low ability



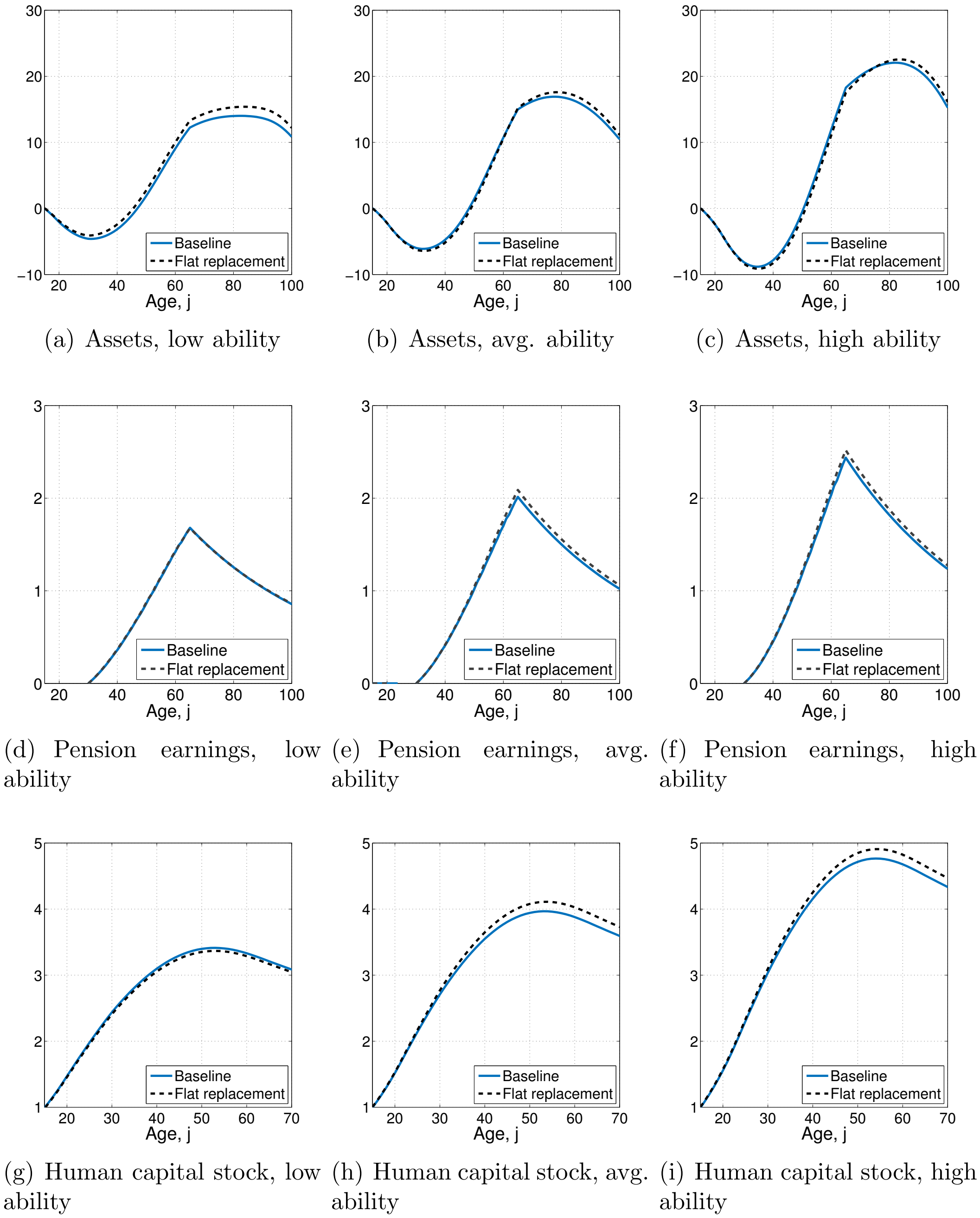
(h) Saving, avg. ability



(i) Saving, high ability

Figure 3: Household consumption, cash-in-hand, and saving over the life cycle by ability level: 2000 birth cohort (productivity de-trended)

Notes: Cash-in-hand includes net labor income, pension benefits, asset income, and bequest.



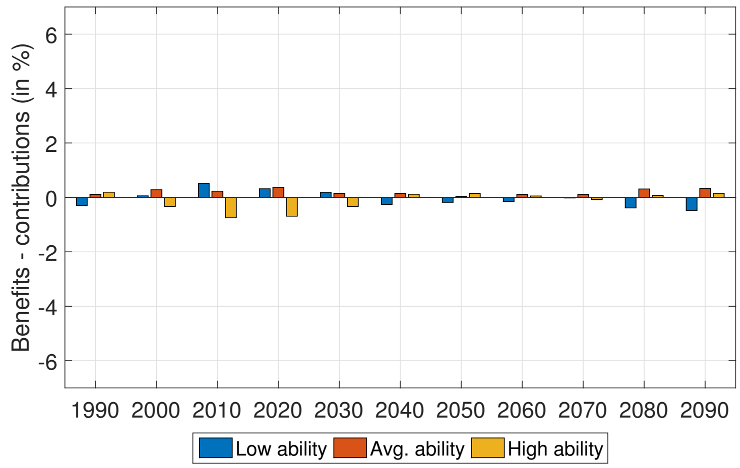
(a) Assets, low ability (b) Assets, avg. ability (c) Assets, high ability

(d) Pension earnings, low ability (e) Pension earnings, avg. ability (f) Pension earnings, high ability

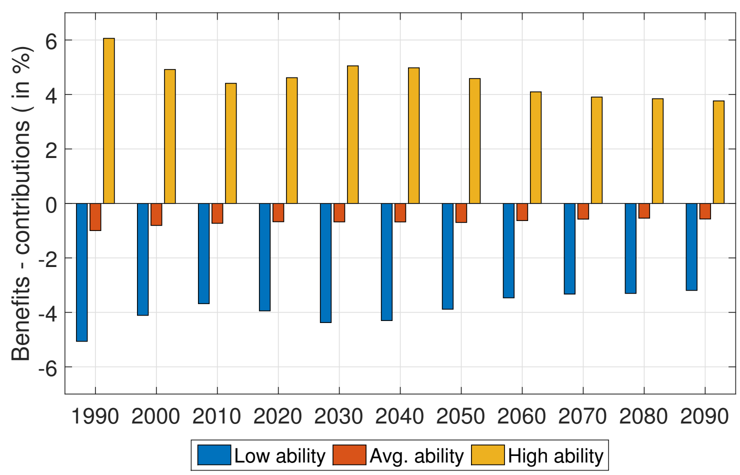
(g) Human capital stock, low ability (h) Human capital stock, avg. ability (i) Human capital stock, high ability

Figure 4: Assets, pension earnings, and human capital stock over the life cycle by ability level: 2000 birth cohort

Notes: Assets and pension earning values are de-trended by the exogenous productivity.

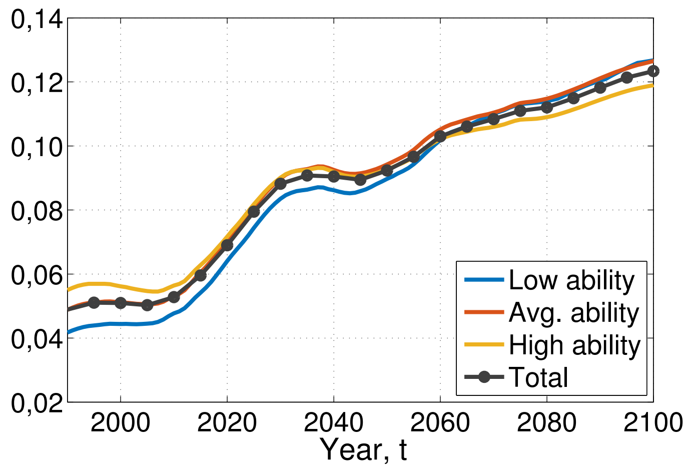


(a) Baseline

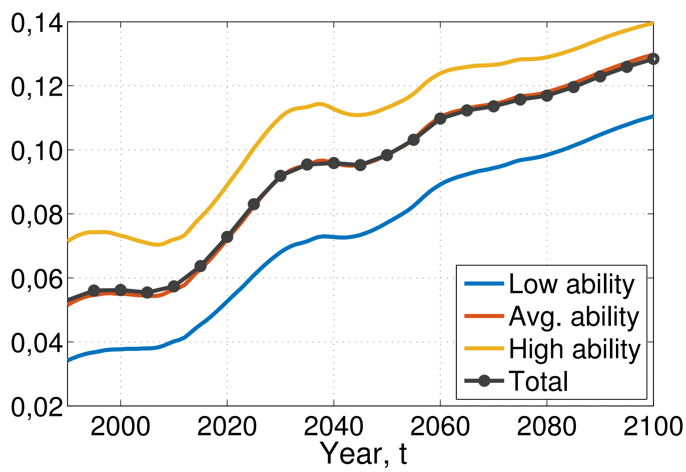


(b) Flat replacement

Figure 5: Benefits minus contributions by ability group (in percentage of the total pension budget): Calendar years 1990-2090

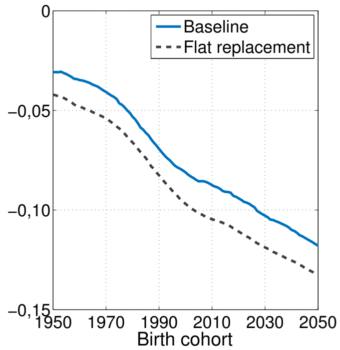


(a) Baseline simulation

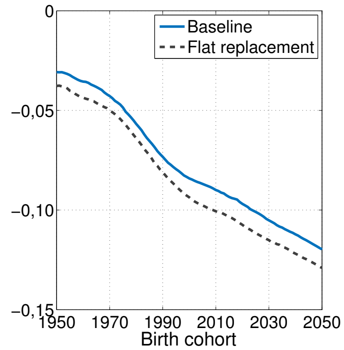


(b) Flat replacement simulation

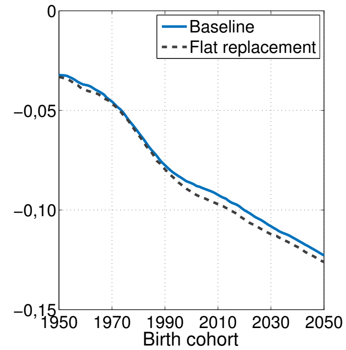
Figure 6: Total pension to output ratio from 1990 to 2100



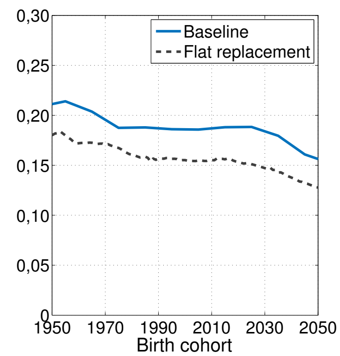
(a) Discounted by the interest rate, Low ability



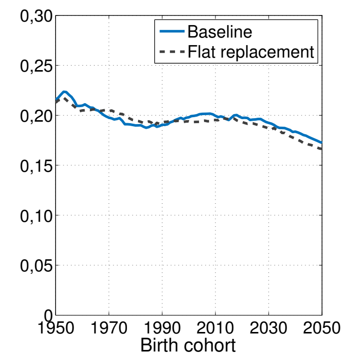
(b) Discounted by the interest rate, Average ability



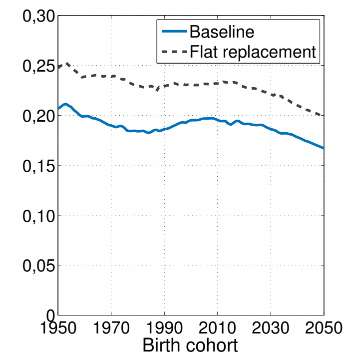
(c) Discounted by the interest rate, High ability



(d) Discounted by the survival probability, Low ability



(e) Discounted by the survival probability, Average ability



(f) Discounted by the survival probability, High ability

Figure 7: Social security wealth as a fraction of present value of labor income by alternative discounting factors: 1950-2050 birth cohorts

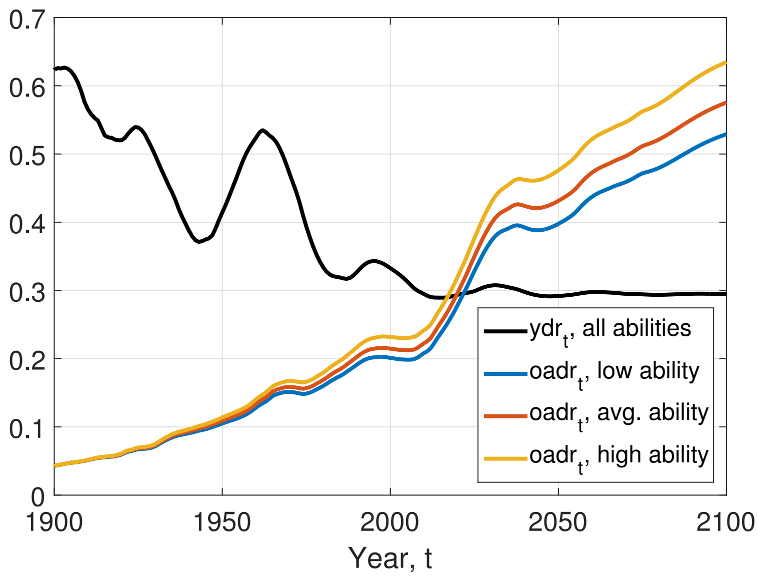
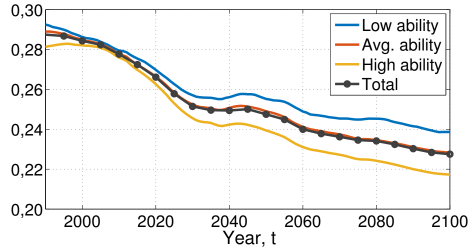
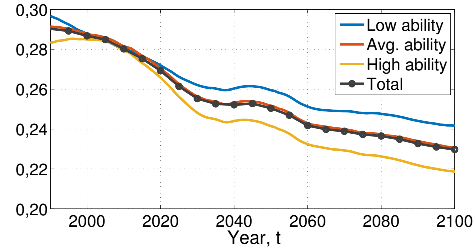


Figure 8: Demographic dependency rates: 1900-2100.

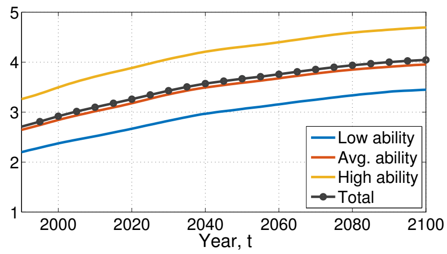
Notes: For convenience we have defined youth between age 0 and 14, adulthood between age 15 and 65, and old-age above age 65.



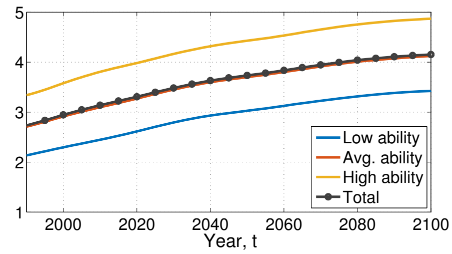
(a) Avg. working time by adults, baseline



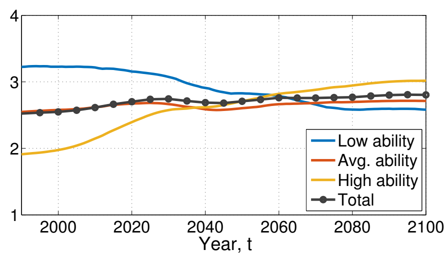
(b) Avg. working time by adults, flat replacement



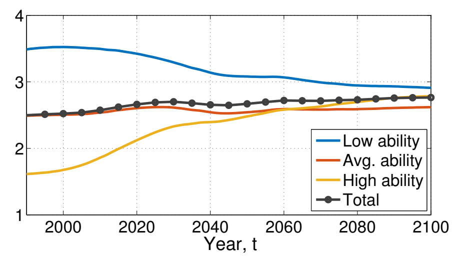
(c) Avg. human capital employed, baseline



(d) Avg. human capital employed, flat replacement



(e) Capital to output, baseline



(f) Capital to output, flat replacement

Figure 9: Decomposition of the output per capita by ability level: Period 1990-2100.

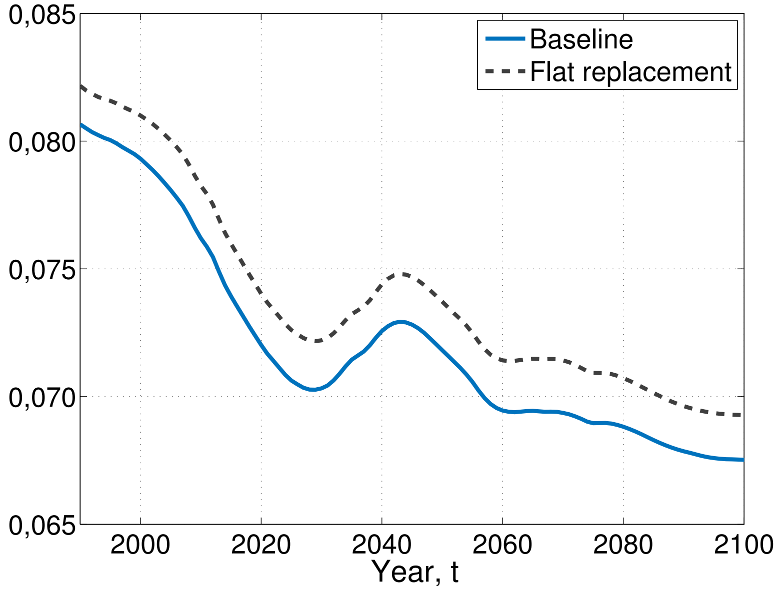
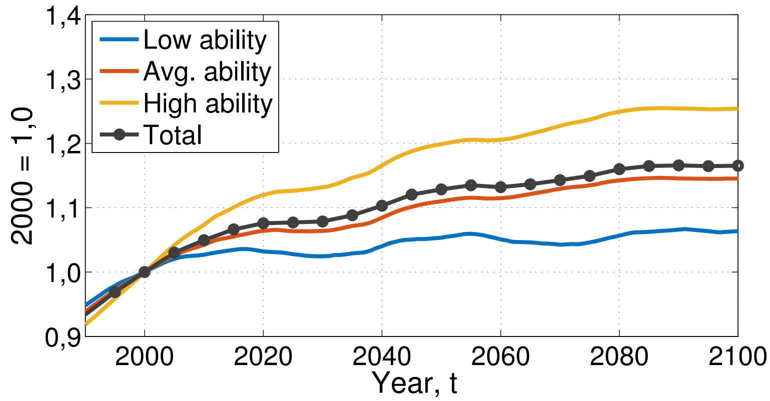
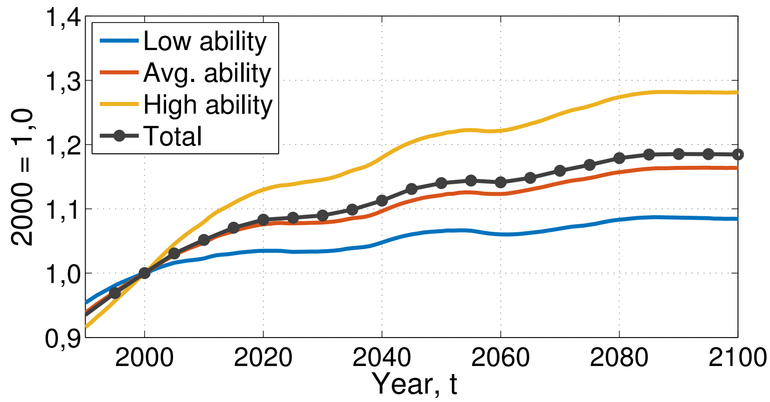


Figure 10: Interest rate



(a) Baseline



(b) Flat replacement

Figure 11: Income per capita (productivity de-trended): Period 1990-2100

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Table 1: Ability and frailty distribution of a new born: $G(\theta, \mu)$

Ability	$\varphi(\theta)$	Frailty $LE(\mu)$		
		$LE(1)$	$LE(2)$	$LE(3)$
Low	0.15	$\frac{1}{6}$	$\frac{1}{12}$	$\frac{1}{12}$
Average	0.16	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{12}$
High	0.17	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{6}$

Table 2: Parametric components of the pension systems

Case	Retirement incentives $\lambda(J_R)$	Replacement rate $\psi(p)$	Capitalization index I_t
Baseline (US)	$\begin{cases} 1+\lambda_p(J_R-J_N) & \text{if } J_R < J_N, \\ 1+\lambda_r(J_R-J_N) & \text{if } J_R \leq J_N \end{cases}$	$\begin{cases} 0.90 & \text{If } p \leq \frac{\bar{y}_t}{6}, \\ 0.32+\frac{0.58}{6}\frac{\bar{y}_t}{p} & \text{If } \frac{\bar{y}_t}{6} < p < \bar{y}_t, \\ 0.15+\frac{1.60}{6}\frac{\bar{y}_t}{p} & \text{If } \bar{y}_t < p < 2\bar{y}_t, \\ \frac{3.40}{6}\frac{\bar{y}_t}{p} & \text{If } 2\bar{y}_t < p. \end{cases}$	$\bar{y}_{t+1}/\bar{y}_t$
Counterfactual	$\begin{cases} 1+\lambda_p(J_R-J_N) & \text{if } J_R < J_N, \\ 1+\lambda_r(J_R-J_N) & \text{if } J_R \leq J_N \end{cases}$	$\psi = 0.385$	$\bar{y}_{t+1}/\bar{y}_t$

Table 3: Early retirement penalties and delayed retirement credits

Year of birth (s)	Credit per year λ_r	Penalty per year† λ_p	Normal retirement age J_N
1917-24	3.0%	6.6%	65
1925-26	3.5%	6.6%	65
1927-28	4.0%	6.6%	65
1929-30	4.5%	6.6%	65
1931-32	5.0%	6.6%	65
1933-34	5.5%	6.6%	65
1935-36	6.0%	6.6%	65
1937	6.5%	6.6%	65
1938	6.5%	6.6%	65+2/12
1939	7.0%	6.6%	65+4/12
1940	7.0%	6.6%	65+6/12
1941	7.5%	6.6%	65+8/12
1942	7.5%	6.6%	65+10/12
1943-54	8.0%	6.6%	66
1955-59	8.0%	6.6%	$66+\frac{2}{12}(s-1954)$
1960-	8.0%	6.6%	67

Source: US Social Security Administration (SSA). Notes: † The yearly penalty rate three years before the normal retirement age is 5%.

Table 4: Model economy parameters

Parameter	Symbol	Value
Preferences		
Intertemp. elasticity of substitution	σ	0.500
Consumption weight	ϕ	0.401
Subj. discount factor	β	1.000
Returns to scale in education	γ	0.650
Initial rate of return to education	φ	{0.15;0.16;0.17}
Human capital depreciation	δ_h	0.008
Technology		
Capital share	α	0.33
Capital depreciation rate	δ	0.05
Labor-aug. tech. progress growth rate	g_A	0.02
Government		
Mandatory retirement age	J_R	65
Weight of current income on pensions	ϱ_j	$\begin{cases} 0 & j \leq J_R - 35 \\ 1/35 & J_R - 35 < j \leq J_R \end{cases}$

Notes: Parameter values withdrawn from Ludwig et al. (2012).

Table 5: Net wealth transfer to present value of lifetime labor income ratio at age 15: Birth cohort 2000

Ability level	Private wealth transfer (WT) I	Social security wealth (SSW) II	Net wealth transfer III=I+II
Baseline			
Low	9.21%	-8.09%	1.12%
Average	8.51%	-8.41%	0.10%
High	8.03%	-8.65%	-0.62%
Flat replacement			
Low	9.58%	-9.65%	-0.07%
Average	8.31%	-9.34%	-1.03%
High	7.23%	-9.06%	-1.83%

Note: Private wealth transfer and social security wealth are defined, respectively, as

$$WT_{2015,15}(\theta) = \sum_{\mu=1}^{3} \sum_{j=15}^{\Omega-1} \frac{\mathrm{tr}_{2000+j,j}(\theta,\mu)}{\prod_{z=15}^{j} R_{2000+z}} \frac{N_{2000+j,j}(\theta,\mu)}{N_{2000+j,j}(\theta)},$$

$$SSW_{2015,15}(\theta) = \sum_{\mu=1}^{3} \left[\sum_{j=66}^{\Omega-1} \frac{b_{2000+j,j}(\theta,\mu)}{\prod_{z=15}^{j} R_{2000+z}} \frac{N_{2000+j,j}(\theta,\mu)}{N_{2000+j,j}(\theta)} - \sum_{j=15}^{65} \frac{\tau_{2000+j} y_{2000+j,j}(\theta,\mu)}{\prod_{z=15}^{j} R_{2000+z}} \frac{N_{2000+j,j}(\theta,\mu)}{N_{2000+j,j}(\theta)} \right].$$