Deriving age-specific death rates from life expectancy forecasts

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Abstract

Predicting the human longevity level in the future by directly forecasting life expectancy offers numerous advantages compared with methods based on extrapolation of age-specific death rates. But the reconstruction of accurate life tables starting from a given level of life expectancy at birth or any other age is not straightforward. Model life table were extensively used in the past for estimating age patterns of mortality in data-poor countries. We propose a new model inspired on indirect estimation techniques that can be used to estimate full life tables given a predicted life expectancy at birth level.

Keywords: Indirect estimation, Life expectancy forecasting, Mortality rates

Introduction

In forecasting demographic processes, such as human mortality, methods involving extrapolation of mortality rates or probabilities are the most common approaches. Stochastic models like those proposed by Lee and Carter (1992) or Cairns et al. (2006) have gained significant popularity and have been extensively used in the last two decades. However, new ideas of focusing only on life expectancy and extrapolation of life expectancy directly have prompted a new field. We can mention here the models introduced by Torri and Vaupel (2012), Raftery et al. (2014) or Pascariu et al.(forthcoming). This type of models is very appealing because they can offer the same level of forecast accuracy in terms of life expectancy but with the advantage of having a simplified formulation. They rely on a measure that incorporates all the variable that influence longevity (lifestyle, access to healthcare, diet, economical status etc.). Furthermore, data highly aggregated by age give valuable information which can be used to tackle the issue of mortality forecasting from a more clearer perspective.

Transformation of life expectancy into mortality rates can be accomplished by exploiting the regularities of age patterns of mortality. In actuarial science the use of life tables, and other models reflecting life contingencies, is motivated by the need to determine insurance and pension risks, net premiums, and benefits. Basically, actuarial methods combine the life table with functions related to an assumed rate of interest. Following the relevance of having a set of age-specific death rates, we propose a method to create such array of values from one available life expectancy. Our method extends the work initiated by Brass (1971) and continued by Murray et al. (2003) and Wilmoth et al. (2012) on indirect estimation techniques of age-patterns of mortality. Relational models were developed for estimation purposes in data poor contexts. As shown here, such models of relations between mortality levels at different ages and for different measures can be used more widely. Specifically, given a predicted level of life expectancy one can decompose it into deaths rates and one-year probabilities. We can use this approach to express the level of mortality as a linear function of life expectancy at birth or above,

$$\log(m_x) = a_x + b_x h + v_x k, \tag{1}$$

where a_x, b_x and v_x are age-specific parameters, h denote the log transform of life expectancy at birth ($h = loge_0$) and k is an estimated correction factor.

We fit this model using a two-step procedure. First we estimate a_x and b_x parameters using the least square approach. In the second step we estimate $v_x k$ term in the spirit of Wilmoth et al. (2012) by computing a singular value decomposition (SVD) of the matrix of regression residuals obtained in the first step.

Figure 1: *Relation between adult mortality and life expectancy in a log-log scale base on 2114 HMD life tables starting from 1950*

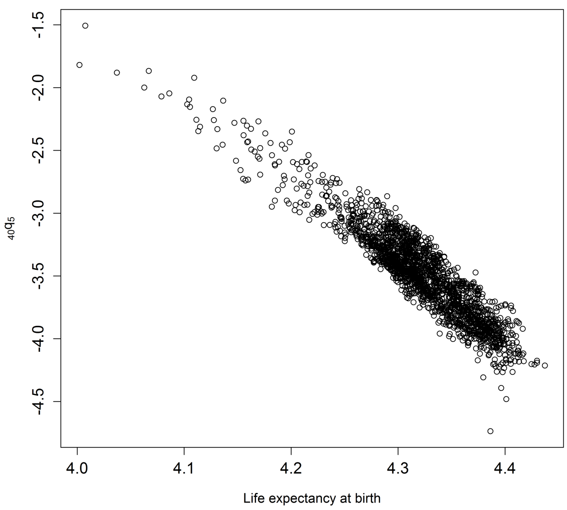


Figure 1 show that a linear model could capture the relationship between adult mortality and life expectancy at birth.

Illustration

In order to test the performance of the method we fit the model using a collection of sex-specific Spanish life tables from the Human Mortality Database (2015) starting from 1980 until 2000 and then estimate mortality rates for 2010 based on a single value of observed life expectancy at birth.

Figure 2: *Estimated pattern of model coefficients over ages*

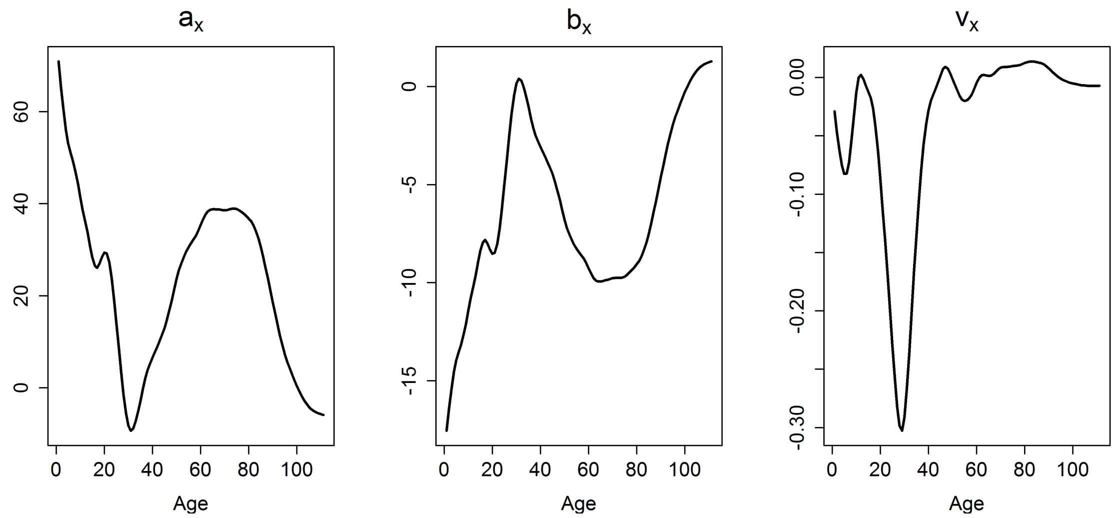
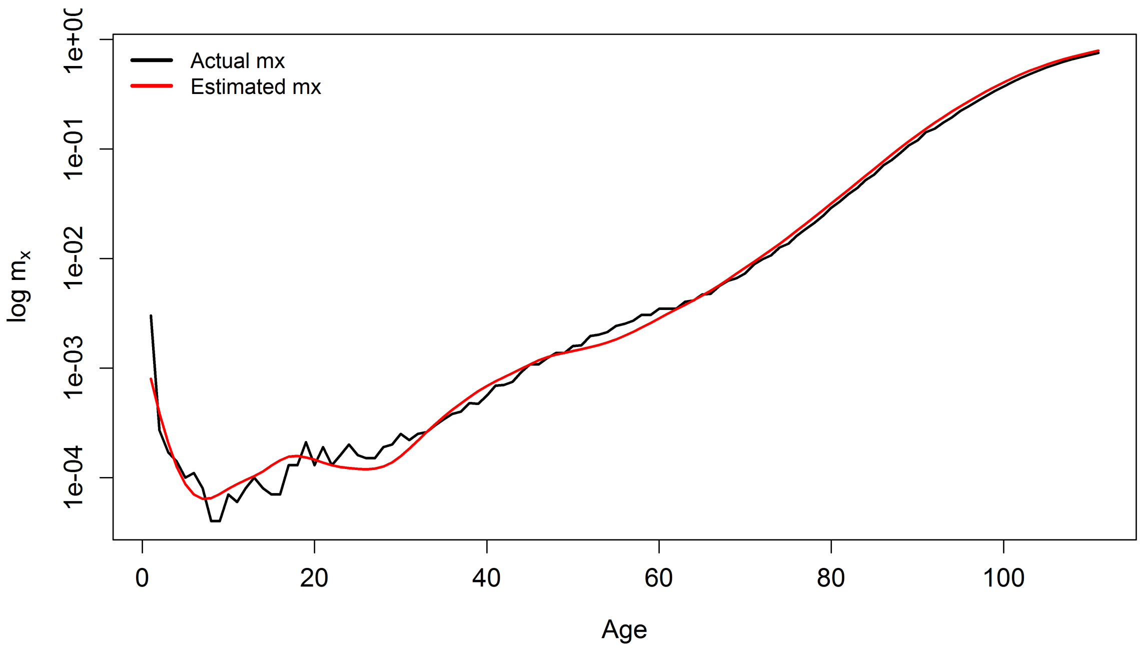


Figure 3 shows a comparison of the Spanish 2010 female death rates against estimated mortality pattern based 2010 life expectancy at birth and 1980-2000 life tables. Depending on the relevance and motivation behind the forecast a different fitting period can be used, like 1950-2010.

Figure 3: *Estimation of 2010 mortality pattern over ages for Spanish females.*



Our most important future step is to find the optimal formulation of the model to estimate age pattern of mortality based on remaining life expectancy at an advanced age, say 65.And also we have strong reasons to believe that better accuracy of the reconstruction can be achieved by using life expectancy at birth and at age 65 into a quadratic relation with the mortality rates.

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