Evolution of premature mortality

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Abstract

We study the changes of premature mortality over time, using a parametric model with response variable the life table distribution at deaths. The model is a mixture of three distributions: one for the infant and child mortality, another for accidental and premature mortality and the last for adult mortality. The main advantages of the model are: the possibility to compute, in explicit form, the three component contributions of life expectancy; the identification of the three modes (one for each function), which helps to split the overall area distribution in the different stages of life. Moreover all parameters have a demographic interpretation. The mixture distribution model is tested using the Swedish raw data from the Human Mortality Database. Our results show that, over time, the premature mortality function becomes flatter and more symmetric, and its mode shifts progressively. This indicates that the accidental mortality has disappeared, while the premature mortality cannot be neglected. We also show that its contribution, both to explain life expectancy and the area of the distribution, decreases in the last century, but in recent years it starts to grow slightly.

1 Introduction

The life table distribution of deaths by age has changed over time and is different across countries, however its elementary structure is preserved. Lexis (1878) divided this distribution in three parts: infant deaths, the normal deaths (a symmetric curve with its maximum on the modal age) and a transition region between youth and adult life, called premature mortality. Clarke (1950) focused on the second part of the distribution and referred to Lexis' normal deaths as senescent and the premature deaths as anticipated. Pearson (1897), took this idea even further and considered this distribution as composed by five functions with different degrees of skewness. In particular he distinguished between infancy and childhood mortality and he identified the youth deaths (accidental mortality) like a symmetric curve with its mode around the age of 25 years; middle life and old age mortality correspond to Lexis' premature and normal deaths respectively. Moreover Pearson asserted that the distribution of adult age can not be symmetric because it depends on the incidence of deaths at earlier ages. Following this consideration we divide the death distribution into three components: infant and child mortality, premature and adult mortality. To fit the adult mortality we use a skew function, so that the premature mortality models the deaths that occur between the youth and the first part of the adulthood: it is the sum of accidental mortality and the excess of deaths that the skew adult curve is unable to estimate.

Different authors have studied the path of the distribution of deaths to understand the evolution of mortality. In fact, since the last century, in European countries, the distribution has experience of the following changes: the infant mortality reduced substantially, without disappearing; the childhood mortality vanished; the accidental hump, very common for male between 20 and 40 years old, now is quite negligible and the adult mortality experienced a shift and a compression. The greater part of researchers focus on the transformations of infant and adult mortality, overlooking what happens to premature mortality. The main reason is the absence of appropriate models to estimate it. In fact the most common mortality models (Gompertz, Makeham, Siler, Kannisto, ecc) do not fit very well this part of the curve, in particular before and during the demographic transition. The most famous model that is able to catch the accidental hump is the Heligman and Pollard model (1980). However some studies show that: i) It is difficult to estimate its parameters due to identification issues (Dellaportas et al, 2001) and, ii) to fit well the accidental mortality for the latest years an extra parameter is required (Heligman and Pollard, 1980). Inspired by the work of Pearson and the ability to account for the accidental hump in the Heligman and Pollard model, we use a newly proposed model which combines those approaches. Our model has less identification problems and it has the advantage to be able to fit the accidental hump when it is reasonably evident.

2 Data

To fit the model we chose the data from the Human Mortality Database. In particular, to test the model we use the raw death counts from Sweden from 1910 to 2011. We chose these country for two reasons. Firstly it has good data, even if we consider the raw historical data (we do not want to test our model with data that are modelled or smoothed themselves). Secondly, Sweden is a furerunner country in terms of reduction in mortality, so if the model fits well its data, we have good chance to get good results also for other countries.

3 Method

We model the life table function of the distribution of deaths (d_x), following Mazzuco et al.(2015) proposal of a new parametric model, to analyse the changes over time of the premature mortality. The selected model is a mixture of distributions, and it is briefly explained below.
To fit infant and childhood mortality a Half Normal (HN) is employed. This distribution, defined only for values greater or equal than 0, has the probability density function (pdf):

$$f_I(x;\sigma) = \frac{\sqrt{2}}{\sigma\sqrt{\pi}} \exp\left(-\frac{x^2}{2\sigma^2}\right) \quad x > 0,$$

where the variable x is the age at death and σ is the shape parameter of the distribution, which is related to the variance. In order to simplify the model, we fix $\sigma = 1$. This choice follows from three considerations: first, when σ is

estimated it gives values close to 1; second, in low infant mortality contexts (very common in Europe since the second half of the last century), the estimation process tends to set big values for σ, so the model is unable to capture the infant deaths (d_0); third, we aim to reduce the number of parameters to avoid identification problems and bad fit.
To capture premature mortality and adult mortality two Skew Normal (SN) Distributions are employed. This type of distributions was developed by Azzalini in 1985 and its pdf is the following:

$$f_{m,M}(x;\xi,\omega,\lambda) = \frac{2}{\omega}\phi\left(\frac{x-\xi}{\omega}\right)\Phi\left(\lambda\frac{x-\xi}{\omega}\right),$$

where $\phi(\cdot)$ is the standard normal cumulative distribution function (cdf), $\Phi(\cdot)$ the standard normal pdf, ξ the location parameter, ω the scale parameter and α the shape parameter. If $\alpha = 0$, a Normal density function is obtained.
Thus, combining the three functions (see Figure 1) with the mixture (or weighting) parameters η and α, we end with a model with 8 parameters:

$$\begin{aligned} f(x,\theta) =&\eta \cdot f_I(x;1) + (1-\eta) \cdot \left[\left(\frac{\alpha}{1+\alpha}\right) f_m(x;\xi_m,\omega_m,\lambda_m) + \right. \\ &\left. \left(\frac{1}{1+\alpha}\right) f_M(x;\xi_M,\omega_M,\lambda_M)\right], \end{aligned}$$

where θ is the vector of 8 parameters, η is the first mixture parameter with value ranging in $[0,1]$ and α is the second mixture parameter which can assume positive values ($\alpha \geq 0$). The parameter α appears in a fraction to allow greater variation in the estimation process. The subscript m indicates the function for premature mortality, while M the function for adult mortality.

All the parameters have a demographic interpretation. The first mixture parameter m is the intensity of infant mortality and it is related to q_0. Moreover, the variance of the Half Normal distribution, $m^2\left(1-\frac{2}{\pi}\right)$, can explain how quickly the child mortality decreases. The second mixture parameter α indicates the importance of the premature mortality (if it is close to 0 we do not have premature mortality and the model is able to explain the young and the adult mortality with only f_M). The three parameters of f_m are: ξ_m (position parameter) which is related to the value of the second mode; ω_m (scale parameter) is related with the variance of the distribution, so if its value is small the premature mortality is concentrated at some ages, while if its value is big, we obtain a vary flat function (it means that the premature deaths affect an ample interval age); if the third parameter λ_m is positive, we obtain a skewness on the right, otherwise the skewness is on the left. We have also three parameters for f_M: ξ_M which is related to life expectancy at birth; ω_M says how much the adult deaths are concentrated around the adult mode; λ_M the parameter of skewness. We expect to observe negative values for this last parameter because, usually, the adult distribution of deaths shows an asymmetry towards the left. Furthermore, if its value is small the adult distribution is close to the symmetry, otherwise we observe a big skewness.

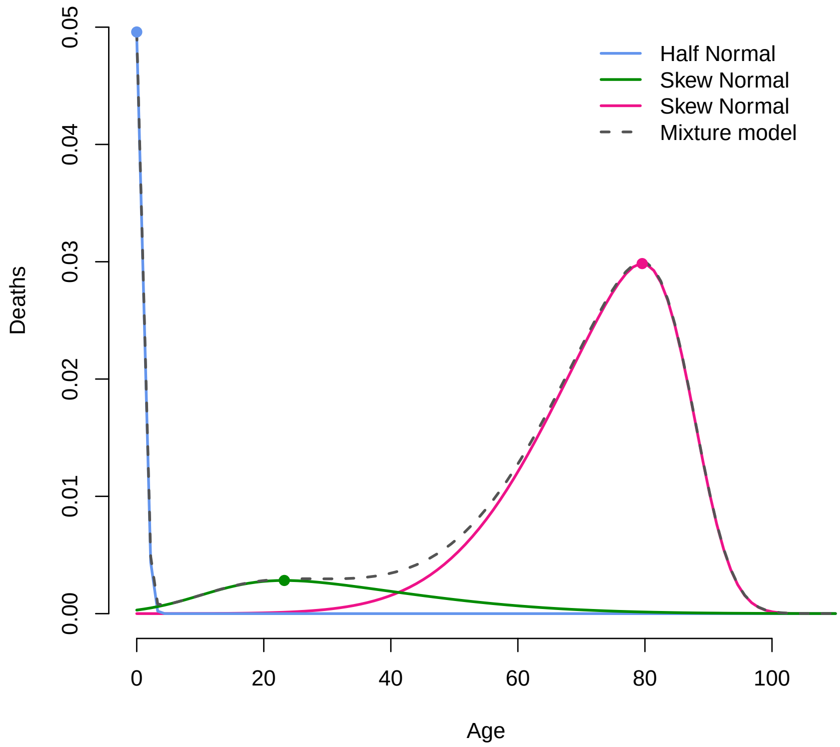

Figure 1: The three different functions of the mixture model (the resulting model is the grey dotted line) and the position of their modes (the solid dots with the respective colours).

Moreover the variance of the two Skew Normal distributions,

$$V_m = \omega_m^2 \left((1-\eta)\frac{\alpha}{1-\alpha}\right)^2 \left(1 - \frac{2\left(\frac{\lambda_m}{\sqrt{1+\lambda_m^2}}\right)^2}{\pi}\right) \qquad \text{and}$$

$$V_M = \omega_M^2 \left((1-\eta)\frac{1}{1-\alpha}\right)^2 \left(1 - \frac{2\left(\frac{\lambda_M}{\sqrt{1+\lambda_M^2}}\right)^2}{\pi}\right),$$

can be interpreted respectively in terms of horizontalization and verticalization (Cheung et al. 2005) of the survival curve. In fact the first variance indicates how many deaths occur before adult mortality, while the second, shows how concentrated the deaths around the modal age of death are.

We use Maximum likelihood to estimate the mixture function. Since we are modelling the probability of the number of deaths, that occur in the age interval $(x, x+1)$, the multinomial distribution is appropriate. The likelihood that follows is:

$$L(\theta; d.x) = \prod_{x=0}^{\Omega} p(x;\theta)^{d.x},$$

where $p(x;\theta)$ corresponds to the number of deaths in the interval x and $x+1$

$$p(x;\theta) = \int_x^{x+1} f_X(t;\theta)\, dt.$$

An advantage of this model is that we can split, in explicit form, the contribution to life expectancy of the three different components: infant, premature and adult mortality. In fact e_0 is the mean of the distribution and it should be divided into the weighted average of the Half Normal distribution and the means of the Skew Normal distributions multiplied by the constants η and α (see Table 1).

Another important measure of longevity used to understand mortality changes is the old modal age at death (Canudas-Romo, 2008; Chueng et al.,2005; Horiuchi et al.,2013; Missov et al., 2015; Bergeron Boucher et al., 2014). For our model it is possible to identify 3 different modes. The Half Normal distribution, describing infant and child mortality, has always its mode at age 0, and its level is very easy to compute because it corresponds exactly to its mean, coinciding with infant mortality at death (d_0). For the other two distributions the values of the modes (m for the premature mortality and M for the adult mortality) are calculated numerically. Whit this three modes it is possible to split the area under the distribution of deaths in five parts (see Figure 2): the infant mortality area, i.e. m because the area of f_I is 1 (we work with distributions , so their integral is 1); the premature mortality area calculated as the integral of f_m between 10 (this age is chosen to delete the infant component) and the last age ω; the adult mortality area, which is $\eta\frac{1}{1+\alpha}$ because the integral of f_M between 0 and ω is 1. We also define the symmetric adult mortality, which is the double integral of f_M between its mode M and ω, and the area of skewness mortality, calculated as subtraction between the integral of f_M and the symmetric adult area (see Table 1).

	e_0		
Mortality	formula	value	Area
Infant	$\eta \int_0^\Omega x \cdot f_I(x;1)\, dx$	$\eta \left(\frac{\sqrt{2}}{\sqrt{\pi}}\right)$	η
Premature	$(1-\eta)\frac{\alpha}{1+\alpha}\int_0^\omega x \cdot f_m(x;\xi_m,\omega_m,\lambda_m)\, dx$	$(1-\eta)\frac{\alpha}{1+\alpha}\left(\xi_m + \omega_m \frac{\lambda_m}{\sqrt{1+\lambda_m^2}}\sqrt{\frac{2}{\pi}}\right)$	$A_m = (1-\eta)\frac{\alpha}{1+\alpha}\int_{10}^\Omega f_m(x;\xi_m,\omega_m,\lambda_m)\, dx$
Adult	$(1-\eta)\frac{1}{1+\alpha}\int_0^\Omega x \cdot f_M(x;\xi_M,\omega_M,\lambda_M)\, dx$	$(1-\eta)\frac{1}{1+\alpha}\left(\xi_M + \omega_M \frac{\lambda_M}{\sqrt{1+\lambda_M^2}}\sqrt{\frac{2}{\pi}}\right)$	$A_M = \frac{1-\eta}{1+\alpha}$
Symmetric	-	-	$A_{sy} = 2\int_M^\omega f(x,\theta)\, dx$
Skewness	-	-	$A_{sk} = \int_{10}^\omega f(x,\theta)\, dx - A_{sy}$

Table 1: Values of the contribution of the tree different function to the calculation of e_0 and integral formula to compute the area of the different component od the deaths distributions.

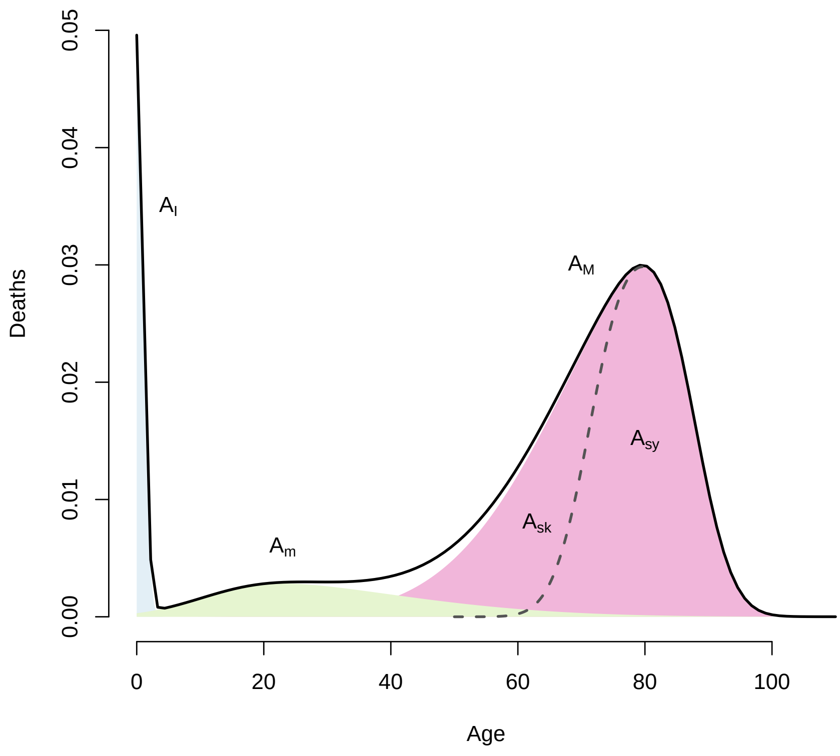

Figure 2: The different areas that compose the death distribution.

4 Results

We estimate the model for Sweden from 1910 to 2010. In Figure 3 we show the fitted function for two different years (red solid line). For each year the model has a good fit and it is also very close to the estimations given by the Human Mortality Database (blue line). We focus our attention on f_m, the dotted green line, because this function is adopted to approximate the premature mortality. In 1935 (during the demographic transition), we can see that its probability is concentrated between 10 and 40 years old. Its mode is around 23 years old, so the premature mortality coincides for the most part with the accidental mortality. In the second graph, classic post-transition path, we observe that f_m is flatter and its probability is spread across the middle life. In fact its mode is close to 50 years of age. In this case, the function estimates deaths that occur almost randomly during the youth and the first part of the adulthood (premature mortality).
In general, looking at the premature mode, we observe a gradual shift of its value, associated with a reduction both of the skewness and the variance (Figure 4). In fact the values of λ_m arrive very near to 0, indicating that the Skew Normal Distribution progressively becomes a Normal Distribution. The decrease of ω_m (and, then, of the variance) is explained considering that at the beginning, f_m fits the accidental mortality and also the premature one, so it requires a big variance because it covers a large age interval (youth and first part of adulthood). With the disappear of accidental mortality, a small variance is necessary because now the function models only the premature mortality.

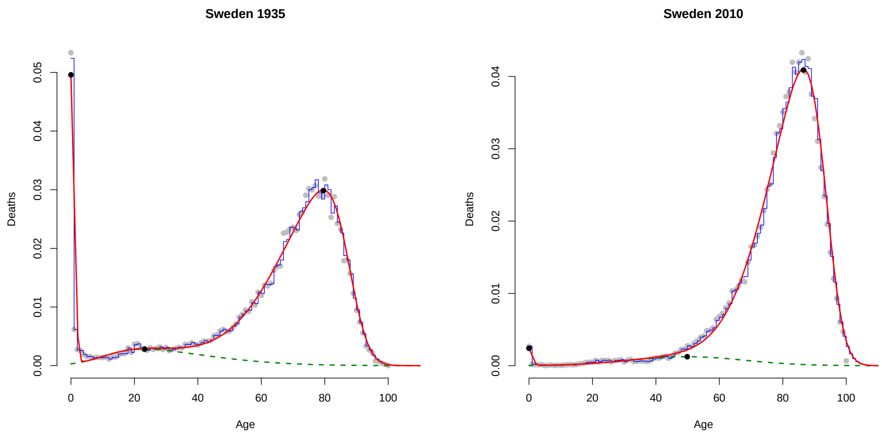

Figure 3: Model fit comparison between two years in Sweden. The row data are plotted in grey, the smoothed data calculated by the Human Mortality Database are the blue curves, the mixture model is the read curve and the green dotted curve is f_m with its mode.

We compute the decomposition of e_0. The results are showed in Figure 5. In the graph we can see the contribution of the three parts of the model, t.i e_{0I}, e_{0m} and e_{0M}, summed progressively: e_{0I}, $e_{0I} + e_{0m}$, $e_{0I} + e_{0m} + e_{0M} = e_0$. The distance between the curve is the values of the single contribution, while the value of the curve is the overall amount. Again we focus on the second

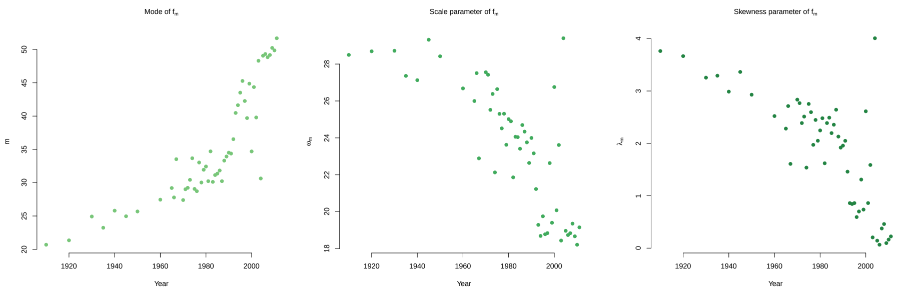

Figure 4: Mode (m), scale parameter (ω_m) and skewness parameter (λ_m) of the premature mortality function f_m.

curve, related to the premature mortality. We can see that its contribution reduced between 1930 and 1950, and, then, it becomes quite constant, without disappearing, like happened to infant mortality contribution. However, in the last few years (1990-2011) it seems to increase slightly, perhaps due to greater incidence of some diseases that affect population during these ages.

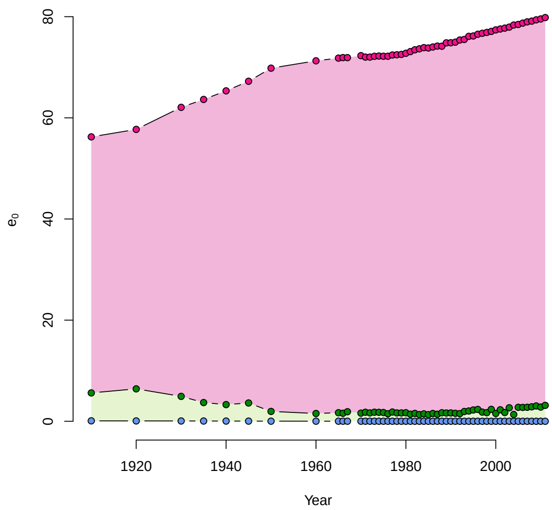

Figure 5: The contribution of the three functions for the calculation of e_0: in pink adult contribution, in green the premature one, in light blue the infant and child one.

Finally, in Figure 6, we show the areas of the integrals of adult mortality and A_{sy}, while Figure 7 displays A_m. As we expect the area of adult mortality covers most of the part of the distribution (it is close to 1). We also note that its value grows rapidly during the first years and then slows down. Figure 6 displays also the values of the symmetric area around the adult mode. As we expected the values of the area increase, indicating both a reduction of accidental mortality and smaller values of skewness (λ_M). Towards recent years the area starts to decrease. This is due to both an increase of skewness (its parameters decrease), and to the presence of the premature mortality (the parameter α is increasing). In fact, the area of premature mortality at the beginning decreases rapidly, and, after a quite stable period, it slightly grows. This result can be interpreted considering that at the beginning, f_m fits the accidental mortality, that gradually disappears in the last century, while from the second half of the considered years, the function catches the premature mortality.

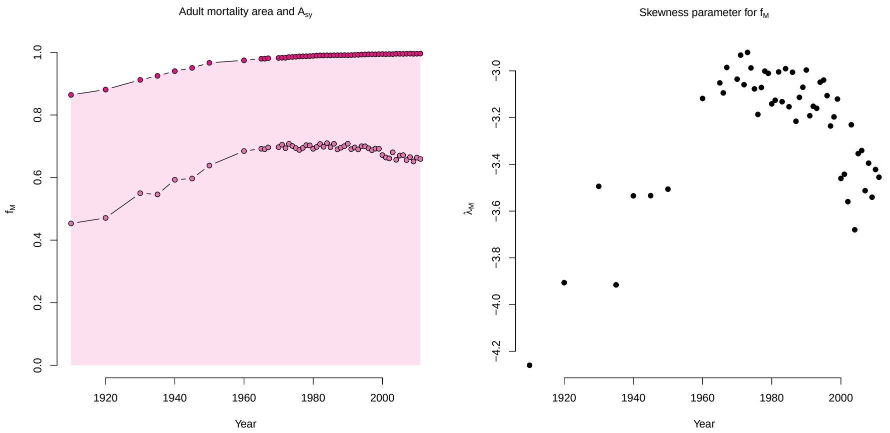

Figure 6: Area of the adult mortality, splitted into its symmetric component and the estimates values of the skewness adult parameter λ_M.

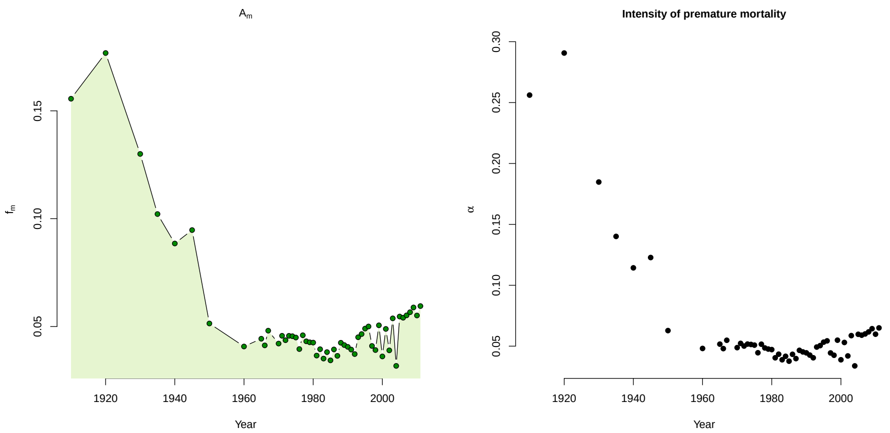

Figure 7: Area of the premature mortality f_m and the path of the parameter α.

5 Conclusions

In conclusion we can say that, during the last century, the accidental mortality is disappearing, making way for the premature deaths across youth and part of

old adulthood.
In this work we show that the contribution of premature deaths is important, first to have a good fit of the model for the data and, second, to understand mortality changes. In fact, even if its contribution to explain life expectancy and its area is not bigger, it gives a greater contribution than infant mortality. Moreover, if the tendency, although light, of premature mortality increase in recent years is confirmed in other countries, it will be necessary to understand what are the causes related to this phenomena.

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