Measuring the Importance of Age

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Abstract

Comparing populations is greatly facilitated by means of simple indices that can readily be calculated for populations with different characteristics. Deriving such indices, we suggest an intuitive method to quantify the impact of age on mortality, where age-dependence can follow any general pattern. The method only requires estimating the constant, age-independent mortality component. Using the constant mortality scenario as baseline, we derive two age-indices that quantify the importance of age. The indices provide a complementary and connected perspective, where both indices can be viewed as two sides of the same coin. We illustrate the workings and utility of the method with various examples. The indices are envisioned to reveal general trends and regularities that may not be (as) apparent from common perspectives. We believe that the method presented here may prove useful, since it is simple, intuitive and closely related to the recently developed pace-shape framework, a new perspective which is starting to reveal interesting results in comparative studies.

Introduction

Populations differ with respect to how long individuals live ('pace') and how mortality changes over age ('shape'). Baudisch (2011) has demonstrated that this new way of thinking about mortality in terms of pace and shape can make a considerable difference to the results in comparative studies that include human and non-human populations. First insights suggest that the pace-shape distinction offers a promising direction (Jones et al., 2014). Related to that approach, here we suggest simple indices that quantify the importance of age in a population. Below we present the method and demonstrate its application with illustrative examples.

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Either age matters, or it does not. If age makes no difference, then mortality at any age x is simply given by

$$\mu(x) = c. \tag{1}$$

Parameter c reflects the constant, positive level of death per unit time. Such an age-independent regime can be interpreted as fully 'pace-driven' or 'pace-governed', since there is no change in mortality, hence the importance of 'shape' is zero. If instead age does matter, then mortality contains an age-dependent component,

$$\mu(x) = g(x) + c, \tag{2}$$

where $g(x)$ is a function of age that takes on real, positive values across some or all ages. Such an age-dependent regime can be interpreted as partially 'pace-driven' and partially 'shape-driven', since there is some change in mortality. Depending on the relative importance of the age-independent vs. age-dependent term, such a mortality pattern will be governed accordingly, more by pace or more by shape.

Lifespan in the first scenario, e_p (subscript p highlighting the connection to pace), simply equals the inverse of c,

$$e_p = \frac{1}{c}. \tag{3}$$

High c implies a short life; low c implies a long life. Lifespan in the second scenario, e_s (subscript s highlighting the connection to shape), is generally shorter than in the first scenario, because an additional, age-dependent term, $g(x)$, poses additional death. For our purposes, we are not interested in any particular pattern of $g(x)$, but just generally acknowledge that

$$e_s < e_p. \tag{4}$$

Comparing these two scenarios allows to quantify the impact of age. If age was unimportant, then both lifespans would be similar to each other,

$$\frac{e_s}{e_p} \approx 1. \tag{5}$$

If instead, age was important, then lifespan in the age-dependent case would be substantially reduced to

$$\frac{e_s}{e_p} << 1. \tag{6}$$

Considering the age-independent scenario as our benchmark case, we can now derive two indices that quantify the impact of age on mortality. The first index compares the *level* of lifespan in the age-dependent scenario with the benchmark case. It hence captures the *age-index of pace*, A_p, and can be calculated as

$$A_p \equiv e_s c. \tag{7}$$

The second index considers the absolute difference in lifespan between the two scenarios. Reflecting relative *change* in lifespan, it captures the *age-index of shape*, A_s, and can be calculated as

$$A_s \equiv \frac{e_p - e_s}{e_p}. \tag{8}$$

Noting that

$$A_s = 1 - e_s c = 1 - A_p \tag{9}$$

we see that both indices capture two sides of the same coin. The age-index of pace A_p reveals to what extend mortality is driven by pace, and the age-index of shape A_s reveals to what extend mortality is driven by shape. Hence, the driving forces of mortality can be decomposed into

$$1 = A_s + A_p. \tag{10}$$

If age does not matter, lifespan is mainly determined by the constant mortality component, and the pace-index A_p is close to one. In this case, the change in lifespan due to age-dependent causes of death is small and unimportant, so the shape-index A_s is close to zero. If instead age does matter, then the level of mortality looses its importance and A_p falls short of one, while change over age gains in importance and A_s substantially exceeds zero. Eventually, if the constant level of mortality is negligible relative to the age-component, then change over age is all that matters and A_s approaches one. In short, the age-indices A_p and A_s reveal to what extend a mortality pattern is governed by pace versus shape over age. If, for example, $A_p = A_s = 50\%$, then age matters just as much as the level of mortality.

The method above can swiftly be applied to compare populations. The absolute difference in the shape-index between two populations A and B is given by $A_s^A - A_s^B$ and the corresponding relative change by

$$\Delta A_s = \frac{A_s^A - A_s^B}{A_s^A}. \tag{11}$$

Analogously,

$$\Delta A_p = \frac{A_p^A - A_p^B}{A_p^A}. \tag{12}$$

Expression 11 (and analogously 12) captures to what extent mortality in population A is governed by shape (and analogously pace) compared to population B. Hence, if $\Delta A_s = 2$ then age was twice as important in population A as in population B. When interpreting quantities 11 and 12, note that the absolute difference in the shape-index has to equal the absolute difference in the pace-index, albeit with opposite signs:

$$A_s^A - A_s^B = (1 - A_p^A) - (1 - A_p^B) = -(A_p^A - A_p^B). \tag{13}$$

Using (13) in (11), one can verify in this context that

$$\frac{A_s^A}{A_p^A} = -\frac{\Delta A_p}{\Delta A_s}. \tag{14}$$

The balance between the shape-index and pace-index in a focal population translates into the balance of relative differences between the focal population and any other population. Thus, if for example in some population the shape-index is twice as large as the pace-index, i.e. $A_s/A_p = 2$, then the relative change in the pace-index will be twice as big as the relative change in the shape-index, if compared to another population.

Preliminary results

As an application for the indices, we look into the historical time trend across time and countries included in the Human Mortality Database (2015), and add data for hunter-gatherer populations and chimpanzees to provide a biodemographic perspective. These data has been extracted from Gurven and Kaplan (2007). For all examples presented here, we calculated the observed as well as the benchmark case using the Siler-mortality-model (Siler, 1983). This model is expressed by

$$\mu(x) = \alpha_1 \, e^{\beta_1 x} + c + \alpha_2 \, e^{\beta_2 x}. \tag{15}$$

Hence, the benchmark case, e_p rests on the inverse of parameter c and the observed case, e_s, is calculated by an approximation of the integral from 0 to the highest age over the survival function derived from Equation 15. The parameter estimates are based on a Poisson log-likelihood procedure. Models have been fitted for each country and each period separately.

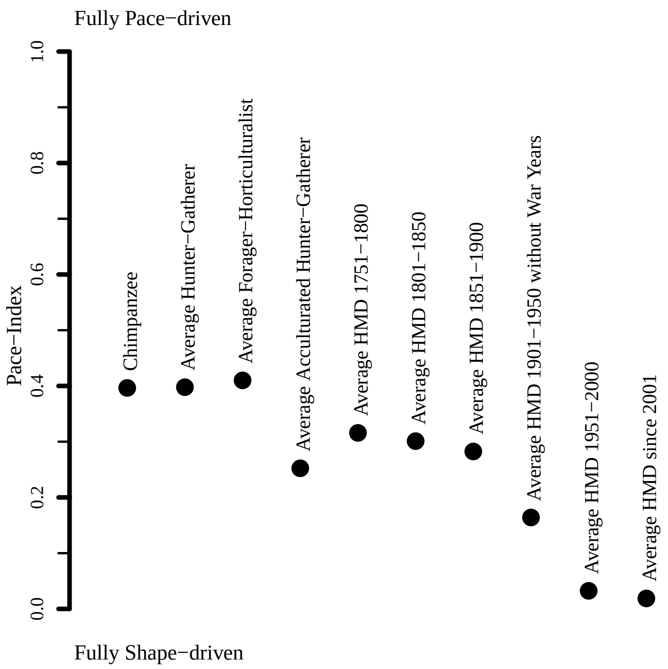

Figure 1. Pace-index, A_p, in a biodemographic perspective: Estimates for chimpanzees and hunter-gatherer population are provided in Gurven and Kaplan (2007). The Human Mortality Database (2015) average is based on all countries with data in the respective time periods and both sexes combined. War Years (1914-1918 and 1940-1944) have been omitted for all countries.

Figure 1 depicts the development of the pace-index, A_p, in a biodemographic perspective. In this graph, a value of 0 corresponds to full shape dependence, meaning age is everything that matters for mortality and a value of 1 refers to full pace dependence, meaning age is not important for the mortality trajectory. The graph clearly depicts the increasing importance of age over human history. Especially when looking at the consistently increasing shape-dependence since the average pattern between 1851 and 1900, it is obvious that increasing environmental control, such as proper hygiene, modern medicine or sufficient food supply, generated the more and more shape-dependent pattern of human mortality, which we observe today.

Future directions

The age-indices developed here allow to exactly quantify the impact of age and are applicable to various topics. Demographic studies on causes of death could benefit from classifying diseases as pace-driven vs. shape driven. The age-indices may also shed new light on studies that look into the impact of natural disasters, pandemics, or other hazardous environments, such as wars. Does an earthquake indeed kill randomly, did the Spanish flu kill randomly, how was the impact of the heat wave in France on different age-groups? How evenly across age groups did the epidemics of chickenpox, cholera or typhus affect people in Europe? How random was death for males vs. females in WWI and WWII? What cause of death could truly be considered fully pace-driven, i.e. independent of age? In general, the index will be useful for comparative studies in biodemography. For example, to sensibly compare death patterns of wild vs. captive animals in studies of senescence, it is important to quantify the impact of the overall level of death in the environment, i.e. pace, vs. the actual age-pattern of mortality that can be assigned to senescence , i.e. shape. Similarly, for modeling purposes in life history biology and ecology, it is important to quantify the error in calculating demographic parameters when assuming constant mortality in a population, where no lifetable is available, but good reasons exist that mortality may indeed be age-dependent. This will have major implications for projecting population dynamics.

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