Examining the Stable Regional Population in Italy using Limit Matricies

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Abstract

The increase of international migration to European Countries has created renewed interest in the overall effects of migration on the distribution of internal population. Using information on inter-regional migration to create a transitional matrix, I aim to find a limit matrix that presents the stable distribution of national population at the regional level. Using this general approach, I aim to decompose the total limit Matrix by age, under the understanding that migratory events are linked closely with specific periods in the life course. After testing the methods with appropriate simulated data, I will apply my techniques to data from the Italian National Institute of Statistics (Istat). Ultimately, my results will provide insight on stable regional populations in Italy by age.

1 Background

While one of the three fundamental components of population change, along with fertility and mortality, the field of migration remains chronically underrepresented within demographic studies. While migration research suffers from incomplete and discordant data, increasing human mobility, both internally and internationally, has increased the importance of migration in when considering population change. The recent influx of international migrants to Europe has raised public interest in the social and economic impacts of migration on receiving countries, increasing the importance of understanding how the aggregation of migratory events into and within a country change the overall population distribution. The ultimate goal of this research will be to find the stable ratio of Italian provincial populations to each other. As briefly outlined below, I will apply matrix diagonalization techniques to a transition matrix of current Istat data to find a limit matrix that reveals these stable proportions.

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2 Approach

I aim to use matrix diagonalization techniques similar to the simplified version below. Ultimately, the methods will be adapted based on the results of simulated data and the data as acquired from Istat.

Given a 2x2 matrix T identifying the change in population proportions at places a and b from time n to 1:

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} a_{a\to a} & a_{a\to b} \\ b_{b\to b} & b_{b\to a} \end{bmatrix} * \begin{bmatrix} a_n \\ b_n \end{bmatrix}$$

The equation can be rewritten to represent the transitions between a and b in n years as:

$$\begin{bmatrix} a_n \\ b_n \end{bmatrix} = \begin{bmatrix} a_{a\to a} & a_{a\to b} \\ b_{b\to b} & b_{b\to a} \end{bmatrix}^n * \begin{bmatrix} a_0 \\ b_0 \end{bmatrix}$$

Using the diagonal matrix D of T's eigenvalues as:

$$D = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

T is estimated as (where X is a matrix):

$$T^n = (XDX^{-1})^n = (XDX^{-1}) * (XDX^{-1}) * ... * (XDX^{-1})$$

By rearranging the combined terms, the equation can be reconsidered as

$$T^n = X * D * (X^{-1}X) * D * (X^{-1}...X) * D * X^{-1}$$

and subsequently reduced to:

$$T^n = X * D^n * X^{-1} \text{ or } T^n = X * \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} * X^{-1}$$

Based on the above simplification, the eigenvalues λ_1 and λ_2 can be estimated given that the determinant of the matrix $(T - \lambda)$ is equal to 0. Assuming the diagonalization of the matrix is possible,

$$\det(T - \lambda) = 0 = \det \begin{bmatrix} a_{a\to a} - \lambda & a_{a\to b} \\ b_{b\to b} & b_{b\to a} - \lambda \end{bmatrix}$$

results in the two eigenvalues, equal to λ_1 and λ_2.

Each column of X is the result of plugging in an eigenvalue, λ, to the above equation

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = (T - \lambda_i) * \begin{bmatrix} X_{1,i} \\ X_{2,i} \end{bmatrix}$$

Given the eigenvales, X may be estimated by concatenating the columns into a matrix with dimensions equal to the initial transition matrix. X^{-1} is calculated:

$X^{-1} = \frac{1}{det(X)} * \begin{bmatrix} X_{2,2} & -X_{1,2} \\ -X_{2,1} & X_{1,1} \end{bmatrix}$

The final ratio of two populations as time approaches infinity can be found as:

$\lim_{x\to\infty} T^n = X * \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} * X^{-1}$

3 Hypotheses and Implications

Given the historical pattern of internal migration in Italy, I expect some regions, specifically traditionally urban areas in the north, to experience great gains in both ratio of the total *and* young adult population. Meanwhile, longterm migratory patterns are likely to result in a concentration of elder populations in rural and southern regions, which have experienced large rates of out-migration in the 20th century.

When comparing the applied model to data points from different time periods, it is likely the limit matricies will reveal information about social and economic characteristics at the time of data collection. For example, I expect to see different long-term results by age group and region when beginning with transition matricies of data from before and after the recent economic recession. While comparisons of past and present data also have the potential to provide information on the effect of current events, this model of matrix diagonalization allows an opportunity to observe the lasting effects of current internal migration trends on the population distribution.