A phase type model of cohabiting union

Jean-Marie Le Goff[1]

[1] Centre for life courses and inequality studies (Lines), University of Lausanne, and Swiss National Center of Competencies in Research Overcoming Vulnerability: life course perspectives (Lives), Switzerland
(E-mail: jean-Marie.LeGoff@unil.ch)

Abstract. In our communication, we propose a phase-type model to analyze the duration of non-conjugal cohabiting unions. This model is a competing risks model with two events that can end a cohabiting union, the marriage with the partner or the separation. We suppose that a non-marital union can be characterized by two hidden phases. The first begins at the start of the union and at each moment, people have the possibility to move to the second phase from which hazard rates of marriage and of separation differ from the first phase. The phase-type model we develop consider that each hazard rate is constant across the time (markovian approach). Investigations made on data of the British 1958 national child study and the 1970 British national study show that the proposed model fits well with data about the first cohabiting union of interviewed people. They however show that processes of marriage and separation differed between the two cohorts.

Keywords: Phase type model, event history analysis family demography, cohabiting unions, event history analysis.

1 Introduction

Extra-marital cohabiting unions spill over in a lot of developed countries during the last decades. Our aim in this paper is to investigate cohabiting unions duration as a phase type model with two kinds of cohabitation termination, the marriage of the couple or its separation (Aalen, 1995; Aalen et al 2008; Linqvist 2000, Lindvisq & Amundrustad, 1998). In the next section (section 2) of this draft, we sketch the interest to develop an approach of cohabiting unions as a phase type model. In the following section (section 3), we present the model we wish to develop. This section is followed by a brief presentation of data we used in this paper, first the 1958 national child study and second the 1970 British national study (section 4). In the section 5, we present and discuss our results and finally conclude.

2. Marriage and cohabiting union formation as composed of different hidden phases

Phase type models are scarce in family demography. The model proposed by Coale and McNeil (1972) to analyse first marriage constitutes an exception. This model can be considered as a kind of Erlang process (Cox, 1962) with a

succession of different social states before the marriage, access to the marriage market at the end of the adolescence, period of partner search, dating and engagement. All these intermediary states, in the model of Coale and Mac-Neil present the peculiarity to be "hidden", because of their difficulty to be delineated in a process of data collection (Coale & McNeil, 1972, Coale, 1977)

According to Coale and McNeil (1972), their model fitted very well marriage distribution in developed countries from the second world war to beginning of the seventies. However in a lot of developed countries, the process of marriage and union formation transformed a lot with the apparition and diffusion of extra-marital unions. Several authors proposed a schema-type of evolution of links between marriage and cohabitations in which each of these two kind of union changed of their meaning (Villeneuve-Gokalp, 1990, Toulemon 1997, Manting, 1996). In a first period, some precursors adopted cohabitation as an alternative to the marriage. These precursors originated from the contestation milieu in youth during the end of sixties and end of seventies that envisaged extra-marital cohabitation as an alternative to the bourgeois marriage (Manting, 1996). Often these precursors were student and lot of them became starting from the seventies managers or exercised intellectual professions. This is however from this milieu that cohabitation diffused to middle and lower classes of societies. During this period of diffusion, cohabitation became a prelude to the marriage, couples experimented the life in common in this kind of union before to eventually get married. In this perspective, marriage remains an engagement to found a family. Starting from the seventies and eighties, several countries experimented an increase in extra-marital births, which indicated that several cohabitors do not wish anymore to marry when they want to have children. Cohabitation replace marriage.

This kind of approach, in which cohabitation is considered to have different meanings is often proposed by authors that are next to the theory of the second demographic transition, in which it is considered that demographic change experimented by developed countries since the end of the baby boom is related to the passage of an industrial to a post industrial society, this transition been characterised at the micro-level by individualization (Lesthaeghe, 1995). An alternative to this approach of cohabitation diffusing along the society is however present in the literature (Reiss & Lee, 1988; Perelli-Harris & Gerber, 2011). In this approach, the choice of cohabitation is constrained by economic reasons. Because marriage is expensive, and because youth, especially young people originated of lower classes experiment precarity on the labour market, couples prefer to cohabit before to marry, sometimes delaying their fertility.

Whatever the characteristics of extra-marital unions in a country or in a social group, chosen or constrained, preceding a marriage or alternative to it, its emergence and its diffusion means a change in the process of marriage as a succession of hidden stages stages, as it was initially proposed in the Coale and McNeil (1972) model. In a country or in a social group in which the extra-marital union is a norm accepted by a large segment of the population a majority of persons, one can consider that the phase of entry on the marriage market, more broadly, on the union market, is a phase that occurs before the

cohabitation. Engagement seems to form a stage that occurs after the entry into cohabitation. Dating is more ambivalent to situate, since it can be considered as a phase that occurs before the cohabitation as well as a phase that occurs after that couples begin to live together. In the case of

3 Model

We propose to develop a model in which cohabiting union is separated into two phases. From each of these two phases, there is the possibility to end this cohabiting union with a marriage or a separation (figure 1). There are three possible events from the first phase of the cohabiting union: marriage, separation, and transition to the second phase of cohabiting union. By hypothesis, each transition rate are constant along the time, which means that the process is markovian. Hazard rates for each of these transition will be respectively noted a, b, and c. From the second phase, there is the possibility to experiment a marriage or a separation with a time constant transition rate of respectively d and e.

Figure 1: phase type model of cohabitation duration

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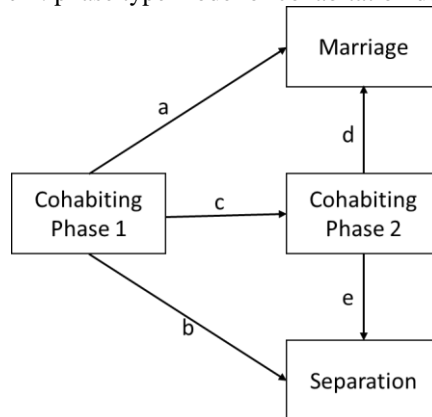
Such an approach into two phases allow to investigate different forms of cohabiting unions. For example, if cohabiting union is a prelude to the marriage in which cohabiting union is considered as a test for each member of the couple on their acceptance to live together, we can suppose that the first phase corresponds to the testing period. In this case, we can expect that hazard rate of marriage a will be low while the hazard rate of separation b will be high. The transition to the second phase corresponds to the end of the testing period, and couples become mature to envisage the marriage. The hazard rate of marriage d becomes during this second phase high while the hazard rate of separation e becomes low.

Other cases can be envisaged. For example, at the beginning of the onset of cohabiting unions in a country, theses unions are adopted by precursors as an alternative to the marriage. We can then consider that the second phase of cohabiting unions, couples adopt this alternative style of life, with thus low hazard rate of marriage d and separation e. The first phase can be considered a phase of experimentation of the cohabiting union, that conduct couples to prefer to be married, to remain cohabitant (transition to the second phase) or to separate. In this case, hazard rate of marriage a, separation b and transition to the second phase of cohabiting union c can be high. If the couple decide to remain cohabitant (transition to the second phase), then hazard rates of marriage and separation become very low

The difficulty of the model is that the transition from the first phase to the second phase is hidden. We do not have any information that allow us to know when a person is in the first phase or in the second phase of cohabiting union. A matricial approach allow to estimate the pdf and densities of transition to marriage and separation. Suppose a markovian infinitesimal matrix, in which each line i and each column j represent the different states possible. In our case, the matrix can be represented by:

$$\begin{bmatrix} -a-b-c & c & a & b \\ 0 & -d-e & d & e \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

0 means that there is no transition possible from the state i to the state j It has been shown that the inverse of this matrix times t allow giving pdf of marriage and separation as well as survival functions in cohabiting unions (Lindvisq & Amundrustad, 1999, Neuts, 1981). We use possibilities of symbolic computations in the software Mathematica to develop formulas (Aalen, 1995). From equations of pdf and survivals, it is possible to compute the density distribution and then the survival function in cohabiting unions. With densities and survival function, likelihood inference to estimate each parameter a, b, c, d, and e can be developed on data with censored values. As usual, in order to avoid negative estimations the hazard rates, we will in fact estimate their logarithm. The command lme in the library stats4 in R allow to make likelihood estimations.

4 data

We used data on first cohabiting unions of two very similar surveys, the National child study and the 1970 British national studies. The first of this survey is a panel that interview more and less regularly all men and women that were borne in a specific week of march 1958. The second survey follows persons born during a specific week of 1970. To be born in 1958 means that those who experiment a cohabiting union in the seventies were something like

early adopters. They represent 45%% of first union. In the second sample, the proportion is of 85% which means that cohabiting union became common for this cohort. In each sample, duration to marriage or to cohabitation were computed. In the case of non observation of any of these events, persons are censored at the moment of their last interview.

4 Results (provisary)

Results presented here were computed on the overall population in each sample. Furthers investigations had shown that results remain very similar when we compare men and women. We first estimated Aalen-Johansen estimators of the cumulative distribution of marriage and separation and survival function in cohabiting union in each sample (figures 2 and 3). Aalen Joahansen are non-parametric methods that generalize the Kaplan-Meier estimator in cases of competing event and multis-states models (Aalen & Johansen, 1978). The package *etm* in R was used to make these non-parametric estimations. Results show that separations are rare in the 1958 cohort while the marriage is more important. A large part of cohabitant remain in this kind of union. In the case of the 1970 cohort, cumulative distribution of each event are more balanced, even if marriage remains more important than separations. Remaining in cohabiting unions are less important in proportion than in the first cohort.

Afterwards, we estimate our phase-type model of cohabiting unions on each sample. Results show very good fit of the model on each sample and give good reasons to validate the hypothesis of a cohabiting union into two phases (figures 2 and 3).
A view on estimated coefficients, allow to understand the processes in cohabiting unions (table 1). In the case of the 1958 cohort the hazard rate of transition from the first phase of cohabiting union to marriage and to the second phase are high, while the transition to separation is rare. When persons are in the second phase, transitions to marriage and to separation are rare. These results correspond well to our hypotheses of cohabiting union as an alternative to the marriage.
In the case of the 1970 cohort, transition to marriage and even to separation are high while the transition to the second phase is scarcer. When people are in this second phase, marriage and separation are rare. Results show then two types of cohabitations. One, corresponding to the first phase, can be considered as a test before the marriage. The second, corresponding to the second phase, correspond give to the cohabiting union as an alternative to the marriage.

Figure 2: Estimation of the model for the cohort 1958
(national child study)
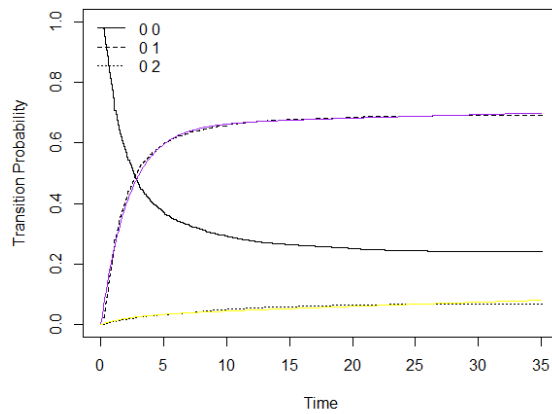
0 0 Survival function. 0 1 cumulative distribution of marriage. 0 2 Cumulative distribution of séparation. In purple: Estimation of the cumulative distribution of marriage with the phase type approach. In yellow: Estimation of the cumulative distribution of divorce with the phase type approach

Figure 3: Estimation of the model for the cohort 1970
(British national study)
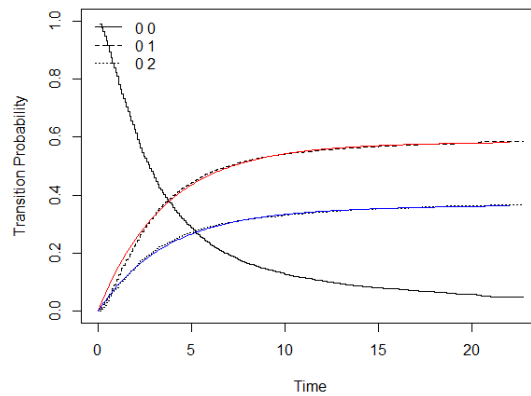
0 0 Survival function. 0 1 cumulative distribution of marriage. 0 2 Cumulative distribution of séparation. In red: Estimation of the cumulative distribution of marriage with the phase type approach. In blue: Estimation of the cumulative distribution of divorce with the phase type approach.

Table 1: Estimated coefficients and hazard rate of each transition

Cohort 58				Cohort 70			
log(hazard)	Estimations	Std.error	hazard rate	log(hazard)	Estimations	Std.error	hazard
a	-1,20	0,02	0,300	a	-1,83	0,02	0,161
b	-4,23	0,09	0,014	b	-2,32	0,02	0,098
c	-1,95	0,04	0,142	c	-3,76	0,13	0,023
d	-5,36	0,13	0,005	d	-4,78	0,66	0,008
e	-5,19	0,09	0,006	e	-4,29	0,32	0,014

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